МАТЕМАТИЧНІ МЕТОДИ, МОДЕЛІ ТА ІНФОРМАЦІЙНІ ТЕХНОЛОГІЇ В ЕКОНОМІЦІ

UDC 519.86
DOI: https://doi.org/10.32782/2708-0366/2023.16.34

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STUDY OF A MARKET MODEL WITH A FIXED DEMAND LINE

ДОСЛІДЖЕННЯ РИНКОВОЇ МОДЕЛІ З ФІКСОВАНОЮ ЛІНІЄЮ ПОПИТУ

In general, the market mechanism is influenced by many factors: tastes and preferences of consumers, interests of sellers, competition between sellers, level of market monopolization, state legislation, seasonal changes. Part is random. All these factors cannot be taken into account. Consider the market mechanism from the point of view of a seller who sells one product. The seller decides what price to set, how much product to offer to consumers. As a result, having sold the product at a favorable price, he can get a bigger profit, or, on the contrary, remain with the unsold product and incur losses. It is obvious that three cases are possible: a shortage of the product, a surplus of the product, and an equilibrium state. For market research, we use such an approach as simulation modeling. The use of simulation models is of great importance for the analysis of economic systems.

Key words: *dynamical system, demand function, offer function, optimization, simulation modeling.*

На ринку є безліч чинників, які впливають на його поведінку: смаки та вподобання споживачів, інтереси споживачів та продавців, конкуренція, монополізація ринку, законодавство у країні, сезонні зміни. Деякі мають випадковий характер. Все врахувати неможливо. Розглянено ринок одного товару із погляду продавця, який його реалізує. При цьому можливі три випадки: дефіцит товару, надлишок товару та рівноважний стан. Побудована модель, ціль якої: визначення оптимального обсягу закупівель, що забезпечує продавцю найбільший прибуток. Також враховано запізнення постачання та інерційність ринку. Для дослідження ринку застосовано такий підхід, як імітаційне моделювання. Застосування імітаційної моделі має велике значення для аналізу економічних явищ. Це дає переваги у порівнянні з виконанням експериментів над реальною системою та використанням інших методів. Проаналізовані моделі Вальраса-Маршалла та павутиноподібна модель. У моделі Вальраса-Маршалла ринкова вартість залежить від попиту та пропозиції, тобто, від потреб і коштів покупців, з одного боку, та від праці і витрат виробників, з іншого. Динамічна модель визначає зміну ринкових чинників у часі. Усі змінні є функціями часу. У павутиноподібній моделі обсяг пропозиції реагує на зміни цін із деяким запізненням. Тоді аналіз моделі ускладнюється. Розмір попиту визначається цінами поточного періоду, а величина пропозиції визначається цінами попереднього періоду, тобто необхідний обсяг товару надходить із запізненням. Вирішуючи завдання пошуку оптимальних обсягів за-

купівель, розглядається ринкова модель без лінії пропозиції. Функція попиту вважається незмінною. Враховується запізнення поставок. Ціну визначає ринок, тобто. за фіксованого обсягу товарів встановлюється ринкова ціна, саме вона забезпечує найбільший прибуток. Змінюючи стратегії закупівель та обсяги замовлень, можна підібрати оптимальну стратегію таким чином, щоб визначити оптимальну лінію пропозиції. Інерційність ринку означає, що у невеликому проміжку часу ціна постійна. Певні рамки обмежують торговця від значного підвищення чи зниження ціни.

Ключові слова: *динамічна система, функція попиту, функція пропозиції, оптимізація, імітаційне моделювання.*

Formulation of the problem. The study of economic phenomena presents presents an interesting and challenging task. The peculiarity is that the study of these processes on real objects can be difficult or even impossible and leads to costs, so you need to look for ways to avoid these difficulties. One way to solve this problem is to estimate unknown quantities through simulation. It is convenient to study many economic processes with the help of models. Considering the problem of finding optimal supplies, we highlight the most important factors: demand, price of goods, volume of purchases. It is necessary to consider processes in time, i.e. in dynamics. Time is discrete. The purpose of building the model is to determine the optimal volume of purchases that provides the seller with the greatest profit. It is necessary to consider processes in time, i.e. in dynamics. Time is discrete. The purpose of building the model is to determine the optimal volume of purchases that provides the seller with the greatest profit. It is also necessary to take into account the delay in deliveries and market inertia. The inertia of the market implies that there are certain limits that limit the manufacturer from a significant increase and decrease in prices. Lowering the price leads to dissatisfaction with competitors, raising the price leads to sanctions from the administrative authorities. If the seller single-handedly changes the price, he may incur significant losses. The delay in deliveries can be caused by the remoteness of the manufacturer from the place of sale, it is always present to one degree or another. Considering the most important factors, we will build a market model to determine how the seller should behave.

Analysis of recent research and publications. The problem of building a market model, modeling and forecasting its development is one of the most important problems of the economy in connection with Ukraine's transition to market relations. Most of the market models were built according to the principle of establishing a competitive equilibrium, the existence of which was declared in the work of Walras [1]. Mathematical substantiation of the Walras hypothesis was carried out in the 1950s in the works of Arrow-Debre [2], McKenzie, Gale, Nikaido. Further work was carried out on the improvement of models and their generalization. These studies are considered quite fully in the monographs of Morishima, Nikaido, Lancaster and other modern authors. Most of these works analyzed the balance of aggregate supply and demand (Market equilibrium) [3; 4]. These market models established a balance between supply and demand, but could not be a market model, since, firstly, there was no competition between both producers and consumers in them, and secondly, the purposefulness of the actions of market participants was not reflected (producers and consumers), which is the basis of competition. The market model should reflect not only the balance between supply and demand, but also the purposefulness of each market participant, taking into account their overall relationship. A vector (multi-criteria) problem of mathematical programming [5] is such a mathematical model that, along with the balance sheet, can reflect the purposefulness of each market participant. To solve this problem, the methods of solving the vector problem, based on the normalization of criteria and the principle of guaranteed result, have been developed [6; 7].

Formulation of the purpose of the article. Study of the impact on the dynamics of the price of goods of random fluctuations in demand, the position of the demand line, the purchase price of goods, the strategy of ordering goods. Statistical evaluation of the seller's profit at a fixed price of goods and a strategy for ordering goods. Determination of the optimal price and order strategy, taking into account the delay. Development of a mathematical model that simulates the market for one product, taking into account random fluctuations and delays, which allows estimating the seller's profit, as well as finding the price of the product and the volume of purchases that provide the seller with the greatest profit.

Presentation of the main research material. *1.Walras-Marshall model.* In the Walras-Marshall model [1; 2], the market price depends on supply and demand, i.e. on the needs and money of buyers, on the one hand, and on the labor and expenses of producers, on the other. In general, the demand function can be presented like this: $Q_i^D = Q_i^D(T; P_1 \ldots P_k; I \ldots)$, where D і Q is the volume of demand for the *i*-th product $(i = 1, 2, \ldots k)$, T are tastes and preferences; $P_1 \ldots P_k$ are the prices of all goods; I – cash income. If all factors except price are assumed to be constant, then we can pass to the demand function as a function of price: $Q_i^D = Q_i^D(P_i)$, which characterizes the dependence of the demand for the i -th product only on its own price. In analytical form: $Q_i^D = a - b \cdot P_i$. A change in the quantity demanded occurs when the price of a good changes, while the dependence of the quantity demanded on the price remains unchanged. If the demand function itself changes as a function of price as a result of non-price factors, such as changes in income or consumer tastes, then demand itself changes. The inverse relationship between price and quantity demanded is called the law of demand. The offer characterizes the willingness of the seller to sell a certain amount of goods at a certain price in a certain period of time. Supply quantity – the quantity of a product that an individual seller or a group of sellers wants to sell on the market per unit of time under certain conditions. The bid price is the minimum price at which a seller is willing to sell a given quantity of a given commodity. The dependence of the volume of supply on the factors determining it is called the supply function. In general, the offer function has the form: $Q_i^S = Q_i^S(L_i; P_1 \ldots P_k; T_i, N; \ldots)$. Q_i^S - is the volume of supply for the *i*-th product $(i = 1, 2, \ldots k)$; L_i - is the nature of the technology used in the production of the *i*-th product; $P_1 \ldots P_k$ - are the prices of goods; T_i - taxes and subsidies established for this product; N – natural conditions. If all factors, except for the price of a given product, are assumed to be unchanged, then from the supply function it is possible to pass to the supply function from the price: $Q_i^S = Q_i^S(P_i)$. Thus, the market model is determined by the supply and demand functions. Market equilibrium is determined by the intersection points of supply and demand lines, which correspond to the price P_E and the volume of goods Q_E, equilibrium volume and equilibrium price. Market equilibrium according to Walras is defined as follows. In a state of equilibrium, the market is balanced, neither sellers nor buyers have incentives to break it. Equilibrium condition: $Q^D(P) = Q^S(P)$. When the price is not equal to the equilibrium price, buyers and sellers have reasons to change the current situation. If the price is slightly above the equilibrium price, then excess supply will put downward pressure on the price. If the price is below the equilibrium price, the quantity demanded will be less than the quantity supplied. The excess demand will put upward pressure on the price. In the first case, pressure will be exerted through the competition of sellers, in the second – through the competition of buyers. Thus, an excess of goods leads to a decrease in price, and a shortage of goods leads to an increase in price. The dynamic model determines the change in market factors over time. All variables are functions of time. The process of changing the supply and demand values over time is described by the equation: $\frac{dP}{dt} = h\left[Q^d(P) - Q^S(P)\right] = h\Delta Q^D(P), h > 0$, $\Delta Q^D(P)$ – excess demand at price P. When $\Delta Q^D(P) > 0$, the market price rises, drops for $\Delta Q^D(P) > 0$. In addition to price changes, other changes can occur that lead to imbalance: changes in prices for other goods, changes in fashion trends, seasonal changes, etc.When non-price factors act on supply and demand, the dependences of supply and demand on price change, i.e. the behavior of buyers or sellers changes. With linear supply and demand functions, their parallel shift occurs. This changes the equilibrium price and affects all other quantities.

2. Delay model. In the cobweb model [3], the volume of supply reacts to price changes with some delay. Then the analysis of the model becomes more complicated. The amount of demand is determined by the prices of the current period, and the amount of supply is determined by the prices of the previous period, because the required amount of goods arrives with a delay. $Q_i^D = Q_i^D(P_t)$, $Q_i^S = Q_i^S(P_{t-\tau})$, where τ – is the delay. The producer determines at time t − τ the volume of supply at time t, focusing on demand and price in period t − τ. Then the price, the magnitude of demand, supply fluctuate around the equilibrium state. For linear supply and demand functions and discrete time t $Q_t^D = a - b \cdot P_t$, $Q_t^S = c + d \cdot P_{t-1}$, the market price level is determined by the equation: $P_t = [P_0 - P_E]\left(\frac{-d}{b}\right)^t$. P_0 - price at the initial moment $t = 0$; P_E - is the equilibrium price at which $Q_i^D = Q_i^S$. The price P_t fluctuates around the equilibrium level P_E. At the same time, the demand and supply schedule takes on a cobweb-like appearance. The stability of the equilibrium depends on the slopes of the supply and demand lines. If the absolute slope of the demand line exceeds the slope of the supply line, the deviation from equilibrium leads to an increase in price and volume fluctuations, further moving the market away from equilibrium. If the absolute slopes of the supply and demand lines are the same, any initial deviation leads to an increase in price and volume fluctuations of the same amplitude around the equilibrium level. If the absolute slope of the supply line is higher than the slope of the demand line, the fluctuations gradually die out, and the disturbed equilibrium is restored. Thus, the delay in supply for price changes leads to instability of the equilibrium.

3. Demand line in case of random fluctuations. Demand characterizes the tastes and preferences of consumers, the willingness to buy one or another quantity of goods. Moreover, demand is determined not only by the tastes of consumers, but also by their capabilities, income of consumers and the price of this product. The volume of demand for a product is the quantity of this product that an individual, a group of individuals or the population as a whole is willing to buy in a unit of time (day, month, year) under certain conditions. The bid price is the maximum price that buyers are willing to pay for a given quantity of a commodity. The demand equation shows the dependence of the volume of demand on the factors that determine it.

$$Qd_0(t) = Qm - a \cdot P(t) \tag{1}$$

Qm – the maximum amount of demand, a – slope coefficient; t – time (discrete); $P(t)$ – price. We must also take into account that the real demand for a particular product changes spontaneously, under the influence of random factors, and include a random variable in the demand line equation. The demand line equation, taking into account random fluctuations, has the form:

$$Qd(t) = Qd_0(t) + \xi(t) \tag{2}$$

$\xi(t)$ – random normally distributed random variable with expectation 0 and variance σ^2.

4. Mathematical model of the market in the absence of a supply line.

Description of variables. Statement of the problem of optimization modelling.

Solving the problem of finding optimal purchase volumes, we will consider a market model in the absence of a supply line. The demand function is considered unchanged and is given by equation (1). Delay in deliveries τ is taken into account. The price is determined by the market. With a fixed volume of goods, the market price is set, it is it that provides the greatest profit. By changing purchasing strategies and order volumes, one can choose the optimal strategy in such a way as to determine the optimal supply line. The inertia of the market means that the price is constant for a short period of time. Certain limits limit the trader from a significant increase or decrease in price. Denote: $Qs(t)$ – sales volume; $Qz(t-\tau)$ is the volume of purchases at the moment $(t-\tau)$; $Qd(t)$ is the volume of demand; *t* is discrete time; $Q(t)$ is the balance of goods at time *t*; Sales volume at time *t*:

$$Qs(t) = \begin{cases} Q(t) + Qz(t-\tau), & Qd(t) \ge Q(t) + Qz(t-\tau) \\ Qd(t), & Qd(t) < Q(t) + Qz(t-\tau) \end{cases} \tag{3}$$

The quantity supplied at time t is the sum of the balance of goods and the supply of goods at that moment. If the volume of demand is greater than the volume of supply, then this means that all goods will be sold, the number of goods sold is equal to the volume of supply. Otherwise, the volume of demand is less than the volume of supply, the number of goods sold is equal to the volume of demand. Quantity of goods in the next step that is left:

$$Q(t+1)=\begin{cases} 0, & Qd(t)\ge Q(t)+Qz(t-\tau) \\ Q(t)+Qz(t-\tau)-Qd(t), & Qd(t)<Q(t)+Qz(t-\tau) \end{cases} \quad (4)$$

Accordingly, if demand is greater than supply, the remainder is 0. Otherwise, the remainder is equal to the difference between the quantity supplied and the quantity demanded. When determining the next volume of purchases, the condition must be taken into account: if there is an unsold balance, then the next volume of purchases is 0.

$$Q(t)\ge Qd(t),\ Qz(t)=0 \quad (5)$$

Profit function: revenue minus order payment, minus storage fee, minus penalty function

$$J(t)=Qs(t)\cdot P(t)-Qz(t-\tau)\cdot P_1-Q(t)\cdot P_2+F(t) \quad (6)$$

Penalty function: a change in price compared to the price of the previous period reduces profit.

$$F(t)=-\frac{\gamma}{2}\left(P(t)-P(t-1)\right)^2 \quad (7)$$

Purpose: to choose the volume of purchases and the price leading to the maximum profit. The search for the maximum $J(t)$ should be carried out taking into account the inertia of the market.

$$J(t)+F(t)\to \max_{P(t),Qz(t-\tau)} \quad (8)$$

Profit optimality criterion. In the study of this criterion, three zones of volumes of offered goods were identified: the zone of shortage of goods, the zone of overstocking of the market, and the zone of dynamic equilibrium. The boundaries separating these zones:

$$Q^{(1)}(t)=\left(R^*\left(Qm-a^*P(i)\right)\right)/(a+R)\ ;$$

$$Q^{(2)}(t)=\left(R^*\left(Qm-a^*P(i)\right)+a^*Qm\right)/\left(2^*a+R\right);$$

$$Q^{(3)}(t)=\left(R^*\left(Qm-a^*P(i)\right)+a^*\left(Qm-a^*P1\right)\right)/\left(2^*a+R\right);$$

When fulfilling the criterion of market optimality, prices and quantities of goods were found that are optimal for each zone. In the area of commodity shortage, the optimal price is determined by the formula: $P^{(1)}(t)=Qm+R^*P(t-1)/(a+R)$

In the area of market overstocking: $P^{(2)}(t)=\left(Qm+R^*P(i)\right)/\left(2^*a+R\right)$

In the area of dynamic equilibrium: $P^{(3)}(t)=\left(Qm-Q(i+1)\right)/a$

The graph of the conditionally optimal price has the form:

In these ranges, there is an optimal price of the goods and the optimal amount of supplies, which ensure the maximum profit of the seller with full satisfaction of consumer demand. Let us now find the optimal price of the goods and the optimal level of delivery of the goods to the market, which ensure the maximum profit of the seller, if the restrictions for the price of the goods in different zones are met: $P_1<P_{\min}\le P(t)\le P_{\max}=Q_m/a$. We will also solve this problem by zones (in zone 1 – shortage, zone 2 – overstocking of the market, zone 3 – balance of supply and demand, i.e. dynamic market equilibrium).

In the zone 1: $0\le Q(t)\le Q^{(1)}(t)$. After substitution $P(t)=P^{(1)}(t)$ into expression for profit $J(t)$ we get : $J(t)=\dfrac{Q(t)^2}{2R}+\left(P(t-1)-P_1\right)\cdot Q(t)+Q^0(t)\cdot\left(P_1-P_2\right)=J^{(1)}(t)$.

As we can see, $J^{(1)}(t)$ increases monotonically with $Q(t)$ according to a linear-quadratic law, reaching its maximum value at the region boundary at $Q(t)=Q^{(1)}(t)$.

Then the volume of additional supply to the market is equal to $Q^z(t)=Q^{(1)}(t)-Q^0(t)$.

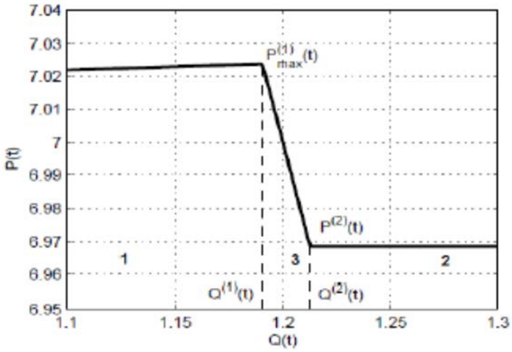

Figure 1. Conditionally optimal price

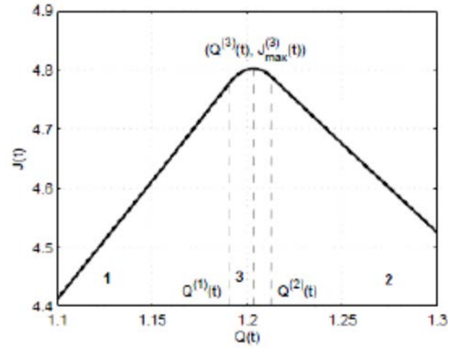

Figure 2. Conditionally maximum profit

In the second zone: $Q(t) > Q^{(2)}(t)$ .After substitution into $J(t)$ for this zone $P(t) = P^{(2)}(t)$, independent of $Q(t)$, we obtain

$$J(t) = \left(Q_m - a\cdot P^{(2)}(t)\right)\cdot P^{(2)}(t) - Q(t)\cdot P_1 + Q^0(t)(P_1 - P_2) - \frac{R}{2}\left(P^{(2)}(t) - P(t-1)\right)^2 = J^{(2)}(t).$$

As we can see, $J^{(2)}(t)$ decreases monotonically with increasing $Q(t)$ according to a linear law, so that it reaches its maximum value in this zone at $Q(t) = Q^{(2)}(t)$ .The volume of additional supply to the market that provides the greatest profit is equal to $Q^z(t) = Q^{(2)}(t) - Q^0(t)$.

In zone 3: $Q^{(1)}(t) \le Q(t) \le Q^{(2)}(t)$. After substituting $P(t) = P^{(3)}(t)$ into $J(t)$ for this zone, we obtain: $J(t) = Q(t)\cdot\frac{Q_m - Q(t)}{a} - Q(t)\cdot P_1 + Q^0(t)(P_1 - P_2) - \frac{R}{2}\left(\frac{Q_m - Q(t)}{a} - P(t-1)\right)^2 = J^{(3)}(t)$

This is a linear-quadratic function. At the maximum point of this function $Q(t) = Q^{(3)}(t)$.

Search for the optimal supply in the presence of delays and random fluctuations in demand. In the presence of delay and random fluctuations in demand, the equilibrium will be disturbed in an unforeseen way. Consider various methods for finding optimal supplies that are beneficial for the seller. When maximizing profit at each moment of time, it should be taken into account that the seller's profit at the moment is affected by the volume of deliveries $Qz(t-\tau)$ ordered earlier. This dependence can be expressed using a function, which is convenient for software implementation and for finding the maximum. We introduce a function that evaluates future profit. The goods arrive at the seller with a delay τ, therefore, knowing the volume of purchases $Qz(t)$ ordered at the moment, we can estimate the profit in the future, simulating sales of goods τ steps ahead. The price is set equal to the market price, the demand is calculated according to equation (1). At each step, using formulas (3), (4), we calculate the number of goods sold, profit, balance of goods. Thus, we estimate the profit in the future $J(t+\tau)$ .The actual profit value will be different, due to random fluctuations in demand. By evaluating the future profit for different volumes of purchases, it is possible to determine the best of them, leading to the greatest profit. The search for the optimal supply is reduced to the search for the maximum profit function in the future on a given interval of acceptable purchase volumes. Various methods can be used for this, but it must be taken into account that the function under study is not continuous. Another way to find the optimal supply is empirical optimization. At each step, the volume of purchases is constant. The search for the optimal supply is carried out for time interval as a whole. That is, a constant optimal supply is determined, leading to the highest profit on average. In this case, you should set a function that determines the dependence of the average profit on the constant value of supplies. The search for the optimal supply means the search for the extremum of this function. Empirical optimization is reduced to a series of tests performed using a model for different supply values. It is necessary to apply an optimization method for search for the maximum average profit, so as not to go through all the options for a given supply interval.

Conclusions. When using a functional relationship to find the optimal purchase, you need to consider which values are variables, which are fixed for a given function, which are present in the model itself, and which are only in the function. When finding the market price, the supply volume is considered known, and the price is profit function argument. When finding the optimal volume of purchases, the price is defined as market (depends on the volume of purchases of the previous period, not this one) and the volume of purchases is an argument of the future profit function. After determining the dependence, the function can be examined for an extremum, which is a convenient way to solve the problem. The profit function is not continuous, which complicates its optimization. To do this, you need to implement one of the algorithms suitable for finding the extremum of non-differentiable functions.

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