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USE OF LINEAR DIFFERENCE EQUATIONS IN ECONOMICS

ВИКОРИСТАННЯ ЛІНІЙНИХ РІЗНИЦЕВИХ РІВНЯНЬ В ЕКОНОМІЦІ

The article examines the use of linear difference equations in modeling economic processes and evaluating their effectiveness for describing market and socio-economic dynamics. Special attention is given to the Evans model, which explains how the market price gradually converges to a new equilibrium value determined by demand and supply conditions. Examples illustrate monotonic convergence and oscillatory approaches to equilibrium, as well as conditions for price stabilization. The cobweb model is also analyzed, highlighting cases where price behavior becomes cyclical or diverges, forming a spiral trajectory. Additionally, a model of unemployment dynamics is presented, resulting in a formula for the steady-state unemployment rate, consistent with classical macroeconomic theories. The study underlines the value of difference equations as a versatile tool for forecasting economic processes, modeling market mechanisms, and identifying long-term trends in economic development..

Keywords: *difference equations, Evans model, spider web model, market equilibrium assessment, demand function, supply function.*

Стаття присвячена застосуванню лінійних різницевих рівнянь у моделюванні економічних процесів та аналізу їх ефективності для опису ринкових та соціально-економічних явищ. Показано, що такі рівняння, навіть за умови обмеженого математичного апарату, дозволяють описувати широкий спектр прикладних завдань та знаходити шляхи їх вирішення. Розглянуто методику розв'язання різницевих рівнянь першого порядку з постійними коефіцієнтами, наведено поетапний висновок загальної формули та її практичне використання для аналізу динаміки економічних показників. Детально проаналізовано модель Єванса, що описує процес поступового наближення ринкової ціни до нового рівноважного значення після зміни умов споживання та пропозиції. Показано, що зміна ціни пропорційна різниці між обсягами поточного споживання та пропозиції, що дозволяє відстежувати шлях ринку до стану рівноваги крок за кроком. Доведено умови збіжності послідовності цін та проаналізовано приклади стабілізації ринкової ціни за різні параметри, що має практичне значення для прогнозування довгострокової поведінки ринку. Розглянуто павутинну модель, що усуває потребу в пропорційному коефіцієнті, але вводитиме нові проблеми, зокрема можливість нескінченних коливань ціни. Проаналізовано випадки, коли ціна стабілізується, коливається навколо рівноваги або рухається по спіралеподібній траєк-

торії, ніколи не досягаючи сталого значення, що демонструє різноманітність сценаріїв розвитку ринку. Наведено приклади з розрахунками, що підтверджують теоретичні положення моделей та демонструють умови стійкого рівноваги. Окремий розділ присвячено моделюванню динаміки рівня безробіття, де на основі різницевих рівнянь визначено, як зміни у рівнях звільнень та працевлаштування впливають на довгострокове рівновагу на ринку праці. Отримано формулу стабільного рівня безробіття, що узгоджується з класичними макроекономічними моделями. Підкреслено значення різницевих рівнянь як ефективного інструменту прогнозування економічних процесів, що забезпечує можливість моделювання складних ринкових механізмів та визначення стійких тенденцій розвитку економіки.

Ключові слова: *різницеві рівняння, модель Єванса, павутинна модель, оцінка ринкової рівноваги, функція попиту, функція пропозиції.*

Formulation of the problem. Economic processes are characterized by constant changes over time, which can occur in discrete rather than continuous steps. Therefore, traditional models based solely on differential equations often fail to adequately describe systems that evolve at fixed time intervals, such as monthly or quarterly market adjustments, investment returns, or employment changes. In such contexts, linear difference equations become an effective mathematical tool for analyzing the dynamic behavior of economic variables, predicting trends, and assessing the stability of equilibrium states.

However, despite their simplicity, the mechanisms of price formation, demand–supply adjustment, and labor market balance still present difficulties in formalization and forecasting accuracy. The problem lies in determining the conditions under which the system reaches equilibrium, oscillates around it, or diverges indefinitely. This study addresses the need to apply and interpret linear difference equations in economic modeling to describe the dynamics of prices, production, and employment, ensuring the mathematical transparency of these processes and enabling the evaluation of long-term economic stability.

Analysis of recent research and publications. Research on dynamic economic processes using linear difference equations has a long-standing tradition rooted in classical price-adjustment frameworks addressing imbalances between demand and supply. Scientists such as Glenn Fulford, Peter Forrester, Arthur Jones, Gerhard Sorger, showed how linear difference equations can be used to model economic and financial processes [1–2]. They explained the relationship between first and higher order difference equations and the dynamics of market variables such as price, profit, investment, demand and supply. Building on this foundation, the Evans-type approach formalizes the path of adjustment toward a new equilibrium: the price change is proportional to the current demand–supply gap, which enables precise statements about the speed and pattern of convergence [3–5]. A closely related strand is the cobweb model, where supply responds to lagged prices; it explains cyclical behavior and highlights parameter thresholds separating stable from divergent regimes. Contemporary publications embed difference equations into broader discrete-time macro frameworks to model inflation dynamics, labor-market flows, and inventories, with emphasis on parameter identification, robustness, and forecast sensitivity. Another active area concerns unemployment models, where linear difference equations deliver transparent steady-state formulas given separation and job-finding rates, clarifying both equilibrium levels and transitional paths. Mankiw combines Keynesian, classical, and neoclassical approaches, showing how they interact in explaining short- and long-run economic fluctuations [6].

Formulation of the purpose of the article. The purpose of the study is to substantiate and demonstrate the potential of using linear difference equations to model economic processes and forecast their dynamics. Investigate well-known models that explain the features of price fluctuations and demonstrate different scenarios of market behavior. Analyze the dynamics of the unemployment rate and determine the conditions for its stabilization based on a mathematical approach.

Presentation of the main material. A great advantage of difference equations is that with their help, using a rather limited circle of mathematical knowledge and skills, it is possible to translate into the language of mathematics (model) and solve a wide class of applied problems, in particular, economic ones. Equation

$$x_n = ax_{n-1} + b \tag{1}$$

is called a first-order linear difference equation with constant coefficients [1]. Here, *a* and *b* are the coefficients of the equation, x_k is the unknown describing the state of the system at the moment *k*.

A characteristic feature of equation (1) is that it is used to describe situations in which the state of the system is completely determined by its state at the previous moment. In this regard, equations of the form (1) are often called *recurrent*.

In order to find the solution to equation (1), we first trace several steps:

$$x_1 = ax_0 + b,$$

$$x_2 = ax_1 + b,$$

substituting for x1 its value from the previous equality, we obtain:

$$x_2 = a\left(ax_0 + b\right) + b = a^2 x_0 + ab + b,$$

$$x_3 = ax_2 + b,$$

and repeating the process, we get:

$$x_3 = a\left(a^2 x_0 + ab + b\right) + b = a^3 x_0 + a^2 b + ab + b,$$

$$x_4 = ax_3 + b = a\left(a^3 x_0 + a^2 b + ab + b\right) + b = a^4 x_0 + a^3 b + a^2 b + ab + b,$$

The trend is clear. An inductive assumption can be made:

$$x_{n-1} = a^{n-1} x_0 + a^{n-2} b + a^{n-3} b + \ldots + ab + b.$$

Induction transition:

$$x_n = ax_{n-1} + b = a\left(a^{n-1} x_0 + a^{n-2} b + a^{n-3} b + \ldots + ab + b\right) + b$$

confirms our assumption

$$x_n = a^n x_0 + a^{n-1} b + a^{n-2} b + \ldots + ab + b. \tag{2}$$

Let's transform equality (2):

$$x_n = a^n x_0 + b\left(a^{n-1} + a^{n-2} + \ldots + a + 1\right) \tag{3}$$

The expression in brackets is the sum of the terms of a geometric progression. Using the formula for the sum of the terms of a geometric progression, from formula (3) we obtain the solution of equation (1).

$$x_n = a^n x_0 + b\left(\frac{1 - a^n}{1 - a}\right), \tag{4}$$

Evans model. The market price of the product was equal to p0. After external circumstances had caused supply and demand in this market to be described by the equations $q^d = a - bp^d$ and $q^s = e - fp^{ds}$ (the coefficients a, b,f are non-negative), Mr. Marshall declared that he knew that the new equilibrium price would be equal to

$a - e / b + f$ [3–4]. In response, Mr. Evans said, "And I know how the market arrives at that price".

Below we present Mr. Evans's reasoning.

A change in market conditions leads to a change in the equilibrium price. Moreover, the change in price is directly proportional to the difference between the volume of current demand and current supply. In mathematical language, this is expressed by the following difference equation:

$$p_n = p_{n-1} + k\left(q_{n-1}^d - q_{n-1}^s\right),$$

where k is the proportionality coefficient determined by non-price factors. Substituting the values q^d and q^s of, we obtain a linear difference equation of the 1st order with constant coefficients.

$$p_n = p_{n-1} + k\left(\left(a - bp_{n-1}\right) - \left(e - fp_{n-1}\right)\right). \quad (5)$$

From here, collecting similar terms, we obtain:

$$p_n = p_{n-1}\left(1 - kb - kf\right) + k\left(a - e\right).$$

Equation (5), according to formula (4), is a function:

$$p_n = \left(1 - kb - kf\right)^n p_0 + k\left(a - e\right)\frac{1 - \left(1 - kb - kf\right)^n}{kb + kf}. \quad (6)$$

In particular, from formula (6) it follows that if the number $\left(1 - kb - kf\right)$ is less than one in absolute value, $k \neq 0$, then when $n \to \infty$ the price will tend to the number indicated by Mr. Marshall: $a - e / b + f$.

Example. Let the demand function be given by the equation $q^d = 10 - p^d$, the supply function by the equation $q^s = 1 + 2p^s$, and the price at the initial moment of time be equal to 2. Then the change in price in this market, according to the Evans model, will be described by the sequence:

$$p_n = \left(1 - k \cdot 3\right)^n 2 + k \cdot 9 \cdot \frac{1 - \left(1 - k \cdot 3\right)^n}{k \cdot 3}.$$

If the market situation does not change for a long time, and $0 < \left|1 - k \cdot 3\right| < 1$, then the price will tend to 3.

In this case,

If $0 < k < 1/3$, then the price will consistently increase from 2 to 3, and if $1/3 < k < 2/3$, the price will approach 3, alternately from above and below.

The cobweb model. Evans' model includes a proportionality coefficient k, which is sometimes difficult to explain [4]. This problem is eliminated with the cobweb model (though new problems arise), which we illustrate with the following situation.

A merchant who carries goods for sale to island X, having learned from the merchants who had been to the island before him that they had sold each box of chocolate for 10 measures of silver, took 10 boxes with him. On the island, it turned out that he could sell all the chocolate for 14 measures of silver per box. Inspired by this price, he brought 14.8 boxes of chocolate the next time. But, unfortunately, he was only able to sell this batch of chocolate for 11.6 measures of silver per box. Therefore, the third time, the merchant took with him only 11.92 boxes of chocolate.

Assuming that the supply of chocolate from the trader and the demand for chocolate from the islanders are linear, we write out the corresponding functions. Next, assuming that no other chocolate traders will go to this island, and that supply and demand will remain unchanged, we determine at what price and at what volume the market will stabilize.

From the conditions it follows that the volume of supply of chocolate on each trip is determined by the price at which chocolate was sold on the previous trip. Hence, the supply function $q_n^s = e + fp_{n-1}$. Substituting the values, we obtain a system of equations that will allow us to determine the coefficients of the supply function:

$$\begin{cases} 10 = e + f \cdot 10, \\ 14{,}8 = e + f \cdot 14. \end{cases}$$

Having solved this system, we obtain that the supply of chocolate from the merchant *t* is given by the function $q_n^s = 1{,}2p_{n-1} - 2$. The validity of this statement can be verified by the data of the 3rd trip: $11{,}92 = 1{,}2(11{,}6) - 2$.

Coefficients of the demand function for chocolate from the islanders $q_n^d = a + bp_n$ are determined by a system of equations:

$$\begin{cases} 10 = a + b \cdot 14, \\ 14{,}8 = a + b \cdot 11{,}6. \end{cases}$$

Solving this system, we get the following result: $a = 38$, $b = -2$.
Therefore, the price at which chocolate is sold on each visit of the merchant, as well as the quantity of goods that he takes to the island each time, are given by the system of equations:

$$\begin{cases} q_n^d = 38 - 2p_n, \\ q_n^s = 1{,}2p_{n-1} - 2 \end{cases}$$

and the initial condition p0 = 10. Since the merchant sells all of the goods each time, the quantity demanded equals the quantity supplied on each trip.

Therefore, the equality $38 - p_n = 1{,}2p_{n-1} - 2$ takes place. Hence, $p_n = -0{,}6p_{n-1} + 20$. The solution to this equation is,

$$p_n = (-0{,}6)^n p_0 + 20\frac{1-(-0{,}6)^n}{1-(-0{,}6)} = (-0{,}6)^n \cdot 10 + 20\frac{1-(-0{,}6)^n}{1+0{,}6}.$$

As n tends to infinity, we find that the price will stabilize at $12, and 13 boxes of chocolate will be sold.

To understand where the name of the model comes from, let's draw supply and demand lines and connect the corresponding dots.

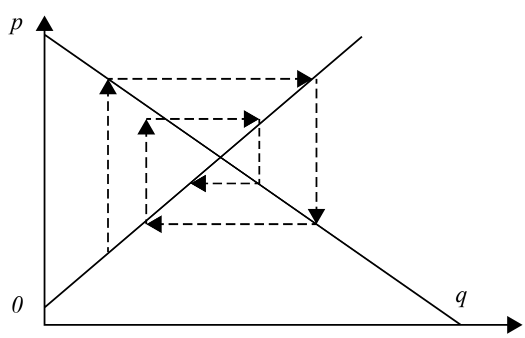

Figure 1. Illustration of a cobweb model of a problem

Source: author's development

We have already mentioned the shortcomings of the cobweb model. As an example, suppose that the demand and supply of chocolate are given by the equations $q_n^d = 38 - 2p_n$ and $q_n^s = 3p_{n-1} - 2$. Then the corresponding difference equation will be $p_n = 1{,}5p_{n-1} + 20$.

His decision:

$$p_n = (-1{,}5)^n p_0 + 20\frac{1-(-1{,}5)^n}{1-(-1{,}5)} = (-1{,}5)^n \cdot 10 + 20\frac{1-(-1{,}5)^n}{1+1{,}5}.$$

Since the sequence $(-1{,}5)^n$ has no limit, we get that the price in such a market will never stabilize.

The corresponding graph has the form of an unwinding spiral.

In order to determine the conditions under which the price on the market in the conditions of the cobweb model stabilizes, we will write the equations of supply and demand in general form: $q_n^d = a - bp_n$, $q_n^s = e + fp_{n-1}$; and then we will write out the difference equation:

$$p_n = \frac{f}{-b}p_{n-1} + \frac{e-a}{(-b)}$$

and solve it:

$$p_n = \left(-\frac{f}{b}\right)^n p_0 + \frac{e-a}{(-b)} \cdot \frac{1-\left(-{f}/{b}\right)^n}{1-\left(-{f}/{b}\right)} = \left(-\frac{f}{b}\right)^n \cdot \left(p_0 + \frac{a-e}{b+f}\right) + \frac{a-e}{b+f}.$$

The price sequence converges, that is, stabilizes at the number $a - e / b + f$, only if the absolute value of the number f / b is less than 1.

In other words, the cobweb model can only predict how the market price will change in the long term if the slope of the demand function is greater in absolute value than the slope of the supply function. In conclusion, we note that the resulting equilibrium price coincides with that obtained based on the Evans model, which is not surprising. It would be strange if this were not the case.

Models of changes in the unemployment rate.

Consider the following problem 1. In country “A” the size of the labor force is constant and equals 2.5 million people, the number of unemployed at the initial moment of time is 120,000. Let 1% of employed people lose their jobs every month, and 19% of unemployed

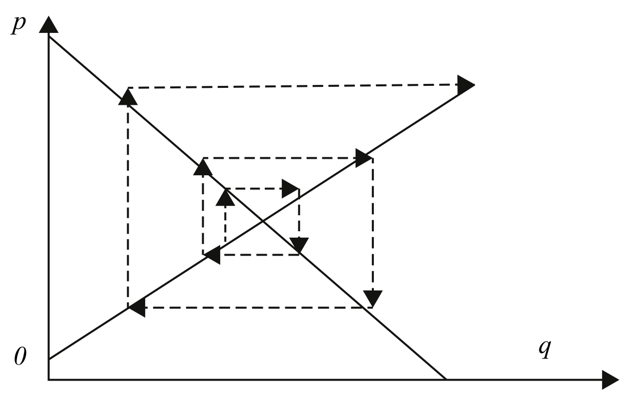

Figure 2. Solution in the form of an unwinding spiral

Source: author's development

people find jobs. How many unemployed people will there be in this country in 9 months? How many unemployed people will there be in this country in many months if the specified conditions do not change?

Let us formulate and solve the problem in general. Let L denote the size of the labor force. It is assumed to be constant. Then, if E_n – is the number of employed, and U_n – is the number of unemployed at the end of the time period with number n, the equality $L = E_n + U_n$ holds.

Note that the number U_n/L is called the unemployment rate.

Let S – be the indicator of the level of dismissal of workers, that is, the share of employed people who lose their jobs in the period under consideration, and let f – be the indicator of the level of employment, that is, the share of unemployed people who find work in the period under consideration. Let's assume that both of these indicators are constant and see that they determine the unemployment rate.

Then, taking into account the fact that $E_n = L – U_n$, we obtain the equation:

$$U_n = U_{n-1}\left(1-f\right)+s\left(L-U_{n-1}\right). \tag{7}$$

Rewriting it in the form (1),

$$U_n = U_{n-1}\left(1-f-s\right)+sL, \tag{8}$$

we obtain a linear difference equation of the first order.

Its solution can be written in the form (4):

$$U_n = U_0\left(1-f-s\right)^n + sL\cdot\frac{1-\left(1-f-s\right)^n}{1-\left(1-f-s\right)}. \tag{9}$$

Assuming that the conditions in the labor market will not change for a long time, from (9) we can (as $n\to\infty$) obtain that

$$U = \frac{sL}{f+s}. \tag{10}$$

From here we can obtain the formula given in Chapter 5 of the famous book by N. Gregory Mankiw "Macroeconomics" [6]: unemployment rate

$$\frac{U}{L} = \frac{s}{f+s}. \tag{11}$$

Let's return to the numerical example, to country "A".

From the conditions: L = 2500000; U_o = 120000; s = 0.01; f = 0.19.

Then, from formula (9) we obtain an estimate of the number of unemployed in this country after 9 months:

$$U_9 = 120000\left(1-0{,}19-0{,}01\right)^9 + 0{,}01\cdot 2500000\cdot\frac{1-\left(1-0{,}19-0{,}01\right)^n}{1-\left(1-0{,}19-0{,}01\right)} = 124329.$$

Assuming that the conditions in the labor market will not change for a long time, we find that equilibrium will be reached in the labor market if the number of unemployed in this country is equal to U = 0,01 · 2500000 · 1/0,19 + 0,01 = 12500, or, in other words, the unemployment rate will stabilize at 12500/2500000 = 0,05 = 5%. This number can be obtained directly from the conditions of the problem by substituting the values s = 0.01 and f = 0.19 into formula (9): 0,01/0,19 + 0,01 = 0,05.

Conclusions. The study has demonstrated that linear difference equations are an effective tool for modeling dynamic economic processes and forecasting their development. A general solution to the first-order equation with constant coefficients was derived and applied to describe the mechanisms of market price formation and the achievement of equilibrium. The Evans model and the cobweb model provided a basis for analyzing the conditions of price trajectory convergence, identifying cases of price stabilization, as well as scenarios of infinite oscillations. The results confirm the versatility of difference equations in forecasting complex economic processes and forming scientifically grounded strategies for economic development.

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