UDC 33.519.8
DOI: https://doi.org/10.32782/2708-0366/2025.23.4

Debela Iryna
Candidate of Agricultural Sciences, Associate Professor,
Kherson State Agrarian and Economic University
(Kherson / Kropyvnytskyi)
ORCID: https://orcid.org/0000-0001-7990-4202

Дебела І.М.
Херсонський державний аграрно-економічний університет
(м. Херсон / м. Кропивницький)

OPTIMIZATION ALGORITHM IN THE MARKOV MODEL OF THE DECISION-MAKING PROCESS

АЛГОРИТМ ОПТИМІЗАЦІЇ В МАРКОВСКІЙ МОДЕЛІ ПРОЦЕСУ ПРИЙНЯТТЯ РІШЕНЬ

Situation in the economic characterized by increasing complexity of tasks, limited reliable data on economic indicators, complex, unpredictable dynamics of processes. Algorithms and tools of subject-object management are becoming relevant because they describe the initial structure of business processes using the categories of economic science, combined with abstract models of the study of complex systems where the results are partly uncertain, partly random. In the theory of Markov processes, a system evolves in time through a sequence of states that depend on previous states and do not depend on the distant past. Management in such systems consists choosing the optimal algorithms for transition from the current state to the state of achieving the goal. However the wide application of the Markov theory is restrained due to the lack of clear rules for calculating the probability of transition between the states of the system under study and the significant uncertainty of the characteristic parameters and optimization criteria. The interpretation of strategic management as a model of system evolution with the Markov property will allow to simplify the process rationalization mechanism in conditions of random changes and partial uncertainty.

Keywords: *algorithm, state, system, management, strategy.*

Ситуація в економічній сфері характеризуються зростанням складності завдань, обмеженістю достовірних даних про економічні показники, складною, непередбачуваною динамікою процесів. В таких умовах інтелектуальні здібності менеджера можуть вступати в протиріччя з тим об'ємом інформації, який необхідно осмислити та опрацювати в ході управління різними технологічними та соціальними процесами. Актуальними стають алгоритми та інструменти суб'єктно-об'єктного управління, що описують оригінальну структуру бізнес-процесів в категоріях економічної науки в поєднанні з абстрактними моделями дослідження складних систем, де результати частково невизначені, частково випадкові, а частково контрольовані. У контексті управління, марковська модель використовується для аналізу стохастичних систем з випадковими переходами між станами. В теорії марковських процесів система розвивається з часом через послідовність станів, де кожен стан залежить від попереднього стану та не залежить від далекого минулого. Управління в таких системах полягає в виборі оптимальних алгоритмів переходу з поточного стану до стану досягнення мети. Але, широке застосування теорії Маркова стримується, через відсутність чітких правил обчислення імовірності переходу між станами досліджуваної системи та суттєвою невизначенністтю характеристичних параметрів та критеріїв оптимізації. Інтерпретація стратетратегічного управління, як моделі еволюції системи з марковською властивістю, дозволить спростити механізм опримізації процесу в умовах випадкових змін. Невизначеність, як категорія, описує якість та детермінованість інформації про стан системи, параметри чи результати впливів факторів. Випадковість передбачає можливість оцінки невідомого параметра імовірнісним розподілом. Переведення оцінки стану системи з невизначеного у випадковий фактично означає оцінку невизначеності стохастичною категорією – імовірністю переходу між станами та уможливлює використання стохастичних моделей оптимізації.

Незважаючи на потужний інструментарій сучасних методів моделювання, дослідження алгоритмів стратегічного управління складними системами залишається актуальним, через низку проблем, які ускладнюють точність та надійність довгострокового прогнозування.

Ключові слова: *алгоритм, стан, система, управління, стратегія.*

Formulation of the problem. The digital transformation of the objects in the social-economics system, new challenges and opportunities of the digital economic creates the basis for revising the traditional structures, means and models of business and organization management existing in practice. Algorithms and tools of subject-object management are becoming relevant, describing the original structure of business processes in the categories of economic science in combination with abstract research models of complex systems, where the results are partly uncertain, partly random, and partly controlled. Strategic management is a decision-making process aimed at achieving strategic goals. The effectiveness of strategic management depends first of all on the possibilities of a formalized description of the future as a model of system evolution. The informative description of such models and algorithms contains a large number of uncertain, non-numerical characteristics and random factors. Therefore there is a need for constant improvement of formalization rules and algorithms. The process of evolution is not isolated in time and space from the external in relation to the very system of the environment and states of nature. It is not possible to determine the influence of the external environment in deterministic indicators. The result of the influence of random factors can only be recorded. The terms "uncertainty" and "randomness" are not equivalent categories from the point of view of formalization. Randomness means that mass random events, phenomena and processes have the property of statistical stability and they are subject to statistical laws. They can be estimated with accuracy to the parameters of statistical distributions accordingly. Uncertainty means the absence of even approximate estimates. It should be noted that the justification of a decision in conditions of uncertainty does not exclude uncertainty itself as a factor. That is one can’t choose a single optimal solution and the accuracy of choosing a solution can’t be as high as in deterministic problems. It is advisable to limit the optimization process to the area of admissible solutions and a set of alternatives that describes the possible states of nature and their impact on the management object.

Analysis of recent research and publications. Economic systems as a rule are quite complex and dynamic structures with partially undefined relationships and parameters. Non-homogeneous information prevails in the information provision of such systems: discrete numerical characteristics and interval values of parameters; statistical quantities and stochastic distributions; descriptive assessments, criteria and limitations. Modern management theory recommends using the combined methodology of the process approach, system and situational analysis [1–3] for the analysis of such systems, without limiting the researcher in the choice of methodological tools. The theoretical basis of strategic management in the postulates of system analysis describes management as an interactive decision-making process [4], which is the result of the process of forecasting, planning, regulating and controlling the activities of the management object. A decision that ensures long-term competitiveness of the object of management in the future can be considered strategic [5, 6]. The solution to the unique problems of management adaptation to changes in the conditions of existence and development of the system is substantiated within the framework of the situational approach [7, 8]. The Markov control model is a mathematical abstraction based on the theory of random processes [9]. In the context of control, the Markov model is used to analyze stochastic systems, where the state of the system evolves in time with random transitions between states. The content of management in such systems consists in choosing the optimal sequence of decision-making stages starting from the current state to the state of achieving the goal. Due to the properties of an abstract representation of complex systems in the form of a set of states and connections between them, Markov chains are often used to model strategic decisions [9, 10]. However, the wide application of the Markov theory is restrained due to the lack of clear rules for calculating the probability of transition between the states of the system under study and the significant uncertainty of the characteristic parameters and optimization criteria. Despite the powerful toolkit of modern modeling methods, the study of control algorithms for complex systems does not become less relevant, due to the rather high risks and uncertainty of long-term forecasts.

Formulation of the purpose of the article. The purpose of the study is an optimization algorithm in the Markov model of the decision-making process.

Presentation of the main material. Strategic management can be described as a process of achieving an optimal balance between the desired and the possible. The implementation of this process takes place over a long period of time. Achieving a strategic goal is a dynamic process that requires constant monitoring and adaptation to changes in the assessment of management results. Dynamics is a reflection of the variation of the states of nature, system parameters, set of criteria for evaluating strategic decisions during the "horizon" of management.

The implementation of the Markov chain model is limited by the requirements for information support which can be formulated in the following list:

– availability of "historical" data. The assessment of transition probabilities between system states is based on the dynamics of previous periods. The number of such periods should be sufficient for a reliable forecast;

– the set of system states must be complete, bounded, and defined within a single state to parameters;

– the set of system behavior alternatives within each state must be complete, bounded, and evaluated within a single state;

– time interval – the "horizon" of management should cover the time interval from the setting of the task to the achievement of the final result;

– the control horizon $t \in [t_0; t_n]$ must be divided into sub-intervals – control steps;

– interval width – control step $\Delta t = [t_i, t_{i+1}]$ should be narrow enough to reflect the dynamics of the control system, but not excessively so as not to capture minor fluctuations.

If the management horizon $T \in [t_0; t_n]$ divide into stages $n_i \in \Delta t_i = [t_i, t_{i+1}]$, which should be considered as separate states of the system $S = \{S_i\}, (i = 1 \div n)$. Everyone state can be defined by a set of alternatives $A = \{A_j\}, (j = 1 \div m)$, and the function of optimality criteria $F = \{F_i(A_j)\}$, then the management process can be described as a dynamic process with step-by-step optimization. It is only necessary to formulate a rule according to which the transition of the system from one state to another is carried out. This rule should be an algorithm for estimating the probability distribution of the system being in a state S_i and transition probabilities between states $[S_i \to S_{i+1}]$. The completed decomposition of the management problem will allow applying Markov processes with discrete states and discrete time to describe the process of strategic decision-making. A key property of Markov that simplifies modeling lies in the assumption that the random process has no "long-term memory" (the probability of transition between states depends only on the probability of the current state and does not depend on the distant past).

As mentioned earlier, uncertainty describes the quality and determinism of information about the state of the system, parameters or results of the effects of factors. Randomness implies the possibility of estimating an unknown parameter with a probability distribution. Transferring the assessment of the state of the system from uncertain to random actually means assessing uncertainty with a stochastic category – the probability of transition between states $S = \{S_i\}$. System states are a set of numerical characteristics of a random process in the control horizon interval $t = [t_0; t_n]$.

The general description of the control process on Markov chains can be described as a functional of a finite set of constraints and parameters:

$$Q\{S; A; P; F\} \tag{1}$$

where: $S = \{S_i\}, i = 0 \div n$ – system states at the moment in time $\Delta t_i = [t_i, t_{i+1}], (\Delta t_i \in [t_0; t_n]);$

$P(S_{ij+1})$ – state transition operator;

$F = \{F_i(A_j)\}$ – the function of the criterion of optimality (reward), which can be interpreted as a criterion of the expected value of the cost of the alternative A_j in condition S_i.

The assumption of model (1) is the stationarity of the set of alternatives within the time interval $\left[t_{i-1};t_i\right]$. At the moment $\Delta t_i=\left[t_i,t_{i+1}\right]$, the value of the state is a multiplicative function of the price of the alternative $F_i(A_j)$ from the plural $A=\left\{A_j\right\}, j=1\div m$ and transition operator.

The transition operator between states can be defined as a limited discrete function of the probability distribution of the system being in the state S_i at the moment of time Δt_i.

In the theory of random processes, the transition between the states of the Markov chain is determined by the matrix of transition probabilities:

$$\pi_{ij}=p\left(S_{i+1}=j|S_i=i\right);\sum\pi_{ij}=1. \quad (2)$$

The components of the matrix can be defined as the posterior probabilities of the alternative $A_j\in\left\{A_j\right\}$ in condition $S_i\in\left\{S_i\right\}$, provided that as a result of the preliminary analysis, the vector of state probabilities is determined $p\left(\left\{S_i\right\}=\left\{p_1,p_2,\cdots,p_n\right\};\sum p_i=1\right)$ and an assessment of alternatives within the state was carried out.

The task of determining the parameters of the initial state $F_0(S_0)$; S_0 and the configuration of the transition operator should be decided at the stage of forming the initial control conditions (t_0).

The object of management is the state of the system S_i, the goal of management is to choose the optimal strategy from a set of alternatives $A=\left\{A_j\right\}$. Then the optimization problem can be formulated as follows: it is necessary to determine the control that transfers the system from the initial state S_0 to the state of achieving the goal S_n, at which the function of the criterion, which is an evaluation of the implementation of alternatives within the state, will acquire an extreme value. Using the principle of R. Bellman [10], the interactive process of optimizing the management strategy can be written as follows:

$$\widetilde{F_i}=\sum_{\{\Delta t_i=S_i\}} exsr\left[\eth\left(S_{0,1}\right)\cdot F_1\left(A_j/S_1\right),\eth\left(S_{1,2}\right)\cdot F_2\left(A_j/S_2\right),\cdots,\eth\left(S_{n-1,n}\right)\cdot\left(A_j/S_n\right)\right] \quad (3)$$

The optimal value of the function $\tilde{F}=\left\{F_i\right\}$ depends not only on the state and the set of alternatives at the moment Δt_i but also from optimal management in the previous period Δt_{i-1}.

The end result is the evaluation of the management process is a recurrent combination of optimization decisions.

Conclusion. The concept of Markov chains is quite a powerful tool for modeling management problems where the outcome is partly random and partly under the control of the manager. Optimization of the strategy is considered a step-by-step controlled process of forming a set of alternatives, in the direction of increasing the stability of the economic system to changes in the external environment. It should be noted that the economic content of strategic management is not determined by the optimization algorithm, but consists in the implementation of the main principles of the efficiency of the development of the enterprise: perspective of decisions; interactivity of the process; priority and implementation of tasks; comprehensive analysis of information about the state of the management system and changes in external conditions. The optimization algorithm should be a tool for analyzing alternative solutions and assessing the achievement of management goals.

References:

1. Tkachova T. (2020) Metodolohichni pidkhody do pryiniattia upravlinskykh rishen na promyslovomu pidpryiemstvi [Methodological approaches to making management decisions at an industrial enterprise]. *Vìsnik Sums'kogo derzhavnogo unìversitetu,* no 1, pp. 199–206. DOI: https://doi.org/10.21272/1817-9215.2020.1-23

2. Kozenkov D., Aloshyna T., Haiduk I. (2022) Metodolohichni pidkhody do pryiniattia upravlinskykh rishen na promyslovomu pidpryiemstvi [Process approach to enterprise management]. *Ekonomika ta suspilstvo,* no 38. DOI: https://doi.org/10.32782/2524-0072/2022-38-67

3. Pererva I. (2021) Perevahy vprovadzhennia protsesnoho pidkhodu do upravlinnia pidpryiemstvom [Advantages of implementing a process approach to enterprise management]. *Ekonomika ta suspilstvo,* no 29. DOI: https://doi.org/10.32782/2524-0072/2021-29-3

4. Yashkina N. (2018) Sutnist ta metodolohiia stratehichnoho upravlinnia: teoretychnyi aspekt [The essence and methodology of strategic management: theoretical aspect]. *Ekonomichnyi prostir,* no 129, pp. 208–215. Available at: https://prostir.pdaba.dp.ua/index.php/journal/article/view/180 (accessed February 19, 2025).

5. Chuhrii H. (2020) Sutnist ta rol stratehichnoho analizu v systemi upravlinnia pidpryiemstvom [Essence and role of strategic analysis in the enterprise management system]. *Scientific Notes of Taurida National V.I. Vernadsky University. Series: Economy and Management.* Vol. 70, no. 2. DOI: https://doi.org/10.32838/2523-4803/70-2-48

6. Debela I. (2023) Kontseptualna model zadachi optymizatsii upravlinnia [Conceptual model of the management optimization problem] *Tavriiskyi naukovyi visnyk. Seriia: Ekonomika,* no 16, pp. 114–118. DOI: https://doi.org/10.32782/2708-0366/2023.16.15

7. Kutashev I. V. (2017) Metodolohichni polozhennia derzhavnoho stratehichnoho upravlinnia rozvytkom sotsialno-ekonomichnykh system [Methodological provisions of the state strategic management of the development of socio-economic systems]. *Investytsii: praktyka ta dosvid,* no 20, pp. 55–58.

8. Korobka S. (2025) Upravlinnia adaptyvnistiu maloho pidpryiemnytstva v umovakh stratehichnoi nevyznachenosti [Management of adaptability in small business under conditions of strategic uncertainty]. *Problemy suchasnykh transformatsii. Seriia: ekonomika ta upravlinnia,* no. 17. DOI: https://doi.org/10.54929/2786-5738-2025-17-04-03

9. Balina O. I., Butsenko Yu. P., Labzhynskyi V. A (2013) Markovska model keruvannia pryrodno-tekhnichnoiu systemoiu [Markov model of management of natural and technical system]. *Upravlinnia rozvytkom skladnykh system,* vol. 16, pp. 175–180. Available at: http://nbuv.gov.ua/UJRN/Urss_2013_16_33. (accessed February 19, 2025).

10. Debela I. (2023) Rekurentnyi alhorytm stratehichnoho kontrolin [Recurrent algorithm of strategic control]. *Tavriiskyi naukovyi visnyk. Seriia: Ekonomika,* no. 16, pp. 274–280. DOI: https://doi.org/10.32782/2708-0366/2023.16.36

Список використаних джерел:

1. Ткачова Т. С. Методологічні підходи до прийняття управлінських рішень на промисловому підприємстві. *Вісник СумДУ. Серія «Економіка».* 2020. № 1. С. 199–206. DOI: https://doi.org/10.21272/1817-9215.2020.1-23

2. Козенков Д., Альошина Т., Гайдук І. Процесний підхід до управління підприємством. *Економіка та суспільство.* 2022. № 38. DOI: https://doi.org/10.32782/2524-0072/2022-38-67

3. Перерва І. Переваги впровадження процесного підходу до управління підприємством. *Економіка та суспільство.* 2021. № 29. DOI: https://doi.org/10.32782/2524-0072/2021-29-3.

4. Яшкіна Н. Сутність та методологія стратегічного управління: теоретичний аспект. *Економічний простір.* 2018. № 129. С. 208–215. URL: https://prostir.pdaba.dp.ua/index.php/journal/article/view/180 (дата звернення 19.02.2025)

5. Чугрій Г. А. Сутність та роль стратегічного аналізу в системі управління підприємством. *Вчені записки ТНУ імені В. І. Вернадського. Серія: Економіка і управління.* 2020. Том 31 (70), № 2. С. 60–65. DOI: https://doi.org/10.32838/2523-4803/70-2-48

6. Дебела І. М. Концептуальна модель задачі оптимізації управління. *Таврійський науковий вісник. Серія: Економіка.* 2023. № 16. С. 114–118. DOI: https://doi.org/10.32782/2708-0366/2023.16.15

7. Куташев І. В. Методологічні положення державного стратегічного управління розвитком соціально-економічних систем. *Інвестиції: практика та досвід.* 2017. № 20. С. 55–58.

8. Коробка С. В. Управління адаптивністю малого підприємництва в умовах стратегічної невизначеності. *Проблеми сучасних трансформацій. Серія: економіка та управління.* 2025. № 17. DOI: https://doi.org/10.54929/2786-5738-2025-17-04-03

9. Баліна О. І., Буценко Ю. П., Лабжинський В. А. Марковська модель керування природно-технічною системою. *Управління розвитком складних систем.* 2013. Вип. 16. С. 175–180. URL: http://nbuv.gov.ua/UJRN/Urss_2013_16_33 (дата звернення 19.02.2025).

10. Дебела І. М. Рекурентний алгоритм стратегічного контролінгу. *Таврійський науковий вісник. Серія: Економіка*. 2023. № 16. С. 274–280. DOI: https://doi.org/10.32782/2708-0366/2023.16.36