UDC 519.8:004.42
DOI: https://doi.org/10.32782/2708-0366/2024.22.5

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SOLVING A LOGISTIC PROBLEM BY DEVELOPING AN OPTIMAL PLAN

ВИРІШЕННЯ ПРОБЛЕМИ ЛОГІСТИКИ ЗА ДОПОМОГОЮ СКЛАДАННЯ ОПТИМАЛЬНОГО ПЛАНУ

The article examines the problem of optimizing logistics processes using the task of developing an optimal plan. A mathematical model of the problem of developing an optimal delivery route has been developed, which is reduced to the classical traveling salesman problem. The proposed model allows taking into account various constraints and optimization criteria typical for real problems. Based on the developed model, a graph was constructed that reflects the structure of the task of delivering windows around the city. To find the optimal route, the MS Excel program was used. The obtained result shows the effectiveness of the proposed approach and its potential for application in other areas where the problem of route optimization is relevant. The conducted research confirms the relevance of the task of creating optimal routes for solving logistics problems. The proposed mathematical model and solution algorithm can be effectively used to optimize delivery processes in various industries. However, there are prospects for further research: expanding the model to take into account dynamic factors such as changes in demand, road conditions and other unpredictable events; integration with geoinformation monitoring systems to obtain more accurate data on distances and travel times; development of interactive web interfaces for convenient use of the developed algorithm by logistics companies.

Keywords: *optimal plan, traveling salesman problem, mathematical model, optimality criteria, objective function, optimal route.*

Ефективна організація логістичних процесів є ключовим фактором успішного функціонування багатьох підприємств. В умовах стрімкого розвитку сучасного бізнесу та зростання конкуренції особливого значення набуває оптимізація маршрутів транспортування товарів, що дозволяє знизити витрати, підвищити якість обслуговування клієнтів та зменшити вплив на навколишнє середовище. Одним із важливих інструментів для розв'язання таких задач є складання оптимального плану перевезень, яке ґрунтується на математичних моделях і сучасних програмних рішеннях. Особливу увагу привертає задача комівояжера, яка є класичним прикладом комбінаторної оптимізації та знаходить застосування в логістиці, плануванні маршрутів, виробничих процесах та інших галузях. У статті досліджено проблему оптимізації логістичних процесів за допомогою задачі складання оптимального плану. Розроблено математичну модель задачі складання оптимального маршруту доставки, яка зведена до *класичної задачі комівояжера. Запропонована модель дозволяє врахувати різноманітні обмеження та критерії оптимізації, характерні для реальних логістичних задач. На основі розробленої моделі побудовано граф, що відображає структуру задачі доставки вікон по місту. Для знаходження оптимального маршруту використано програму MS Excel. Отриманий результат демонструє ефективність запропонованого підходу та його потенціал для застосування в інших галузях, де актуальна проблема оптимізації маршрутів. Проведене дослідження підтверджує актуальність задачі складання оптимальних маршрутів для вирішення логістичних проблем. Запропонована математична модель та алгоритм розв'язання можуть бути ефективно застосовані для оптимізації процесів доставки в різних галузях. Однак, існують перспективи для подальших досліджень, зокрема: розширення моделі з урахуванням динамічних факторів, таких як зміна попиту, дорожніх умов та інших непередбачуваних подій; інтеграція з системами геоінформаційного моніторингу для отримання більш точних даних про відстані та час руху; розробка інтерактивних веб-інтерфейсів для зручного використання розробленого алгоритму логістичними компаніями.*

Ключові слова: *оптимальний план, задача комівояжера, математична модель, критерії оптимальності, цільова функція, оптимальний маршрут.*

Formulation of the problem. The essence of the problem is as follows. One must go around a fixed number of places, starting from the place where you are and finishing your route by returning to the starting place, without visiting anywhere twice. The cost of travel between any pair of specified places or the length of the route between must-see points is known. At the same time, it is not at all obvious that the shortest route will have the minimum cost of travel, which, in turn, really depends on the company, type of flight and mode of transport.

It is required to determine the route or sequence of visiting places that have the minimum total cost among all possible routes.

The evaluation function in this problem is the total length of the full path starting and ending at a certain place, and the constraints are the presence or absence of a flight between individual places on the list under consideration, as well as the need to visit all of these places [1].

Analysis of recent research and publications. Optimization of logistics processes is one of the key tasks of modern business. A challenging task in this industry is to develop optimal delivery routes that minimize transportation costs, increase delivery speed, and enhance customer satisfaction. One tool for solving this problem is the optimal planning problem or the traveling salesman problem, which has a rich history of research and widespread application in economics.

There is a large number of scientific works devoted to the optimization of logistics processes using mathematical models, in particular the traveling salesman problem. These studies cover various aspects of the problem. The traveling salesman's task was first formulated in the 19th century, but its relevance has not diminished to this day [1; 2]. Many scientists have made significant contributions to the development of the theory and practical application of this problem. Karl Menger is one of the first scientists to study the traveling salesman problem in the context of geography. The formal mathematical formulation of the traveling salesman problem was introduced by Harold Kuhn, and George Danzig developed the simplex method of linear programming, which is widely used to solve optimization problems, including the traveling salesman problem. Jack Edmonds made significant contributions to graph theory and combinatorics, which became the basis for the development of effective algorithms for solving the traveling salesman problem. One of the leading modern researchers in the field of logistics systems optimization is Martin Groschel. He developed many efficient algorithms for solving the traveling salesman problem [2].

Formulation of the purpose of the article. The purpose of the article is to study methods for optimizing logistics processes by developing an optimal plan, as well as analyze existing approaches and develop recommendations for improving the efficiency of logistics systems management.

Presentation of the main material. The problem of developing an optimal plan, which is called the traveling salesman problem (TSP) or the problem of reconfiguring equipment, is an example of a classic problem of combinatorial optimization, which looks very simple in its formulation, but requires the most serious efforts to find an exact solution. Although in the general case the traveling salesman problem can be formulated as the problem of finding a closed or open contour (a path passing through all vertices) of minimum length in a graph, a number of features allow it to be classified as a combinatorial optimization problem [3]. This problem is a classic example of a combinatorial optimization problem. Although its formulation as an optimization problem on graphs is quite simple, finding its exact solution is a rather labor-intensive process from a computational point of view. The traveling salesman problem can serve as a test problem for checking computational algorithms for solving combinatorial optimization problems and integer programming in general.

Consider a connected directed graph: $G=(V,E,h)$, in which $V=\{v_1,v_2,\ldots,v_n\}$ is a finite set of vertices, $E=\{e_1,e_2,\ldots,e_m\}$ is a finite set of arcs, $h:E\rightarrow Z_+$ is the weight function of the arcs. For the mathematical formulation of the problem, we denote individual values of the weight function of the arcs as: $c_{ij}=h(e_k)$, where the arc $e_k\in E$ corresponds to an ordered pair of vertices (v_i,v_j). According to the problem statement, individual values: $c_{ij}=h((v_i,v_j))$ are considered as the length of a section of the original graph.

The length of any subset of arcs $E_k\subset E$ in graph G is equal to the sum of the weights of the arcs included in this subset. It is required to determine a subset of arcs that forms a closed path in graph G, passes through each vertex exactly once and has a minimum length [3; 4].

To write the conditions of the traveling salesman problem in the form of a Boolean programming model, we note the following features of the desired route in the graph:

1. Each of the vertices of the original graph must have in the desired route exactly one arc incident to it, which is incoming for this vertex, and exactly one arc incident to it, which is outgoing for this vertex. Otherwise, such vertices will be isolated or dead-end and, therefore, the path will not pass through all the vertices of the original graph.
2. The total number of arcs in the desired path must be exactly equal to n, where n is the total number of vertices in the original graph. Indeed, if some path contains less than n arcs, then it will not pass through all vertices or be cyclic. If the desired path contains more than n arcs, then it will not satisfy the condition of passing each vertex exactly once.
3. The desired path must be a single cycle and must not split into separate cycles with the number of arcs less than n. This condition is of a combinatorial nature.

Let us consider the following Boolean variables x_{ij}, where $x_{ij}=1$ if the arc (v_i,v_j) is included in the desired route of minimum length, that is, the salesman moves directly from the i-th place to the j-th, and $x_{ij}=0$ if the arc (v_i,v_j) is not included in the optimal route, that is, if the salesman does not move directly from the i-th place to the j-th [5, 6].

Then, in the general case, the mathematical formulation of the traveling salesman problem can be formulated as follows [1]:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}c_{ij}x_{ij}\rightarrow\min_{x\in\Delta_\beta}, \tag{1}$$

where the set of admissible alternatives Δ_β is formed by the following system of constraints of the type of equalities and inequalities:

$$\begin{cases}
\sum_{j=1}^{n}x_{ij}=1\ \left(\forall i\in\{1,2,\ldots,n\}\right); & (2)\\
\sum_{i=1}^{n}x_{ij}=1\ \left(\forall j\in\{1,2,\ldots,n\}\right); & (3)\\
u_i-u_j+n\cdot x_{ij}\le n-1\ \left(\forall i,j\in\{1,2,\ldots,n\},i\ne j\right); & (4)\\
x_{ij}\in\{0,1\},\left(\forall i,j\in\{1,2,\ldots,n\}\right); & (5)\\
u_i\in R^1,\left(\forall i\in\{2,3,\ldots,n\}\right). & (6)
\end{cases}$$

This mathematical model of the traveling salesman problem uses auxiliary variables: $u_i\left(\forall i \in \{2,3,\ldots,n\}\right)$, which can take any real values. In this case, restrictions (2) and (3) ensure that the first two conditions specified earlier are met: the desired path must pass through each vertex of the graph exactly once. Constraints (4) ensure that the third of the previously indicated conditions is met: the sought path must not break up into separate cycles. Constraints (5) ensure that the condition is met – variables x_{ij} must take Boolean values, and constraints (6) ensure that the condition is met – variables u_j must take real values. It is easy to show that the total number of constraints (2) – (4) is equal to $2n+(n-1)(n-2)=n^2-n+2$.

Note that those coefficients of the objective function c_{ij} for which the weight function of the edges h of the original graph is not defined or equal to 0, in the mathematical formulation of the problem under consideration (1) – (6) should be set equal to $+\infty$, that is, a sufficiently large positive value.

Let's consider solving the traveling salesman problem using MS Excel using the example of the logistics problem of delivering large-sized goods, for example, windows from a manufacturing enterprise across the city. The problem here is not only to create the shortest route for delivering windows to customers, but also to properly load the machine so that the first to be shipped are those windows that need to be delivered to the first place on the route. Let's draw a graph of window delivery from the enterprise to six points in the city (Fig. 1).

The length of a road section between two adjacent delivery points, expressed in km, is equal to the value of the weighting function for each arc. This value is indicated next to the image of the corresponding arc in the graph.

It is required to find such a complete closed path starting at vertex number 1 and ending at vertex number 6 so that the total length of the path is minimal.

The variables of the mathematical model of this traveling salesman problem are 36 variables: $x_{ij}\left(\forall i,j \in \{1,2,\ldots,6\}\right)$, each of which x_{ij} takes the value 1 if the arc (i,j) is included in the minimum complete path, and 0 otherwise, and 5 auxiliary variables: $u_i\left(\forall i \in \{2,3,4,5,6\}\right)$, which can take any real values. Additionally, for convenience of calculations, the values of the weights c_{ii} should be set equal to some sufficiently large positive number, for example, $c_{ii}=100\left(\forall i \in \{1,2,\ldots,6\}\right)$.

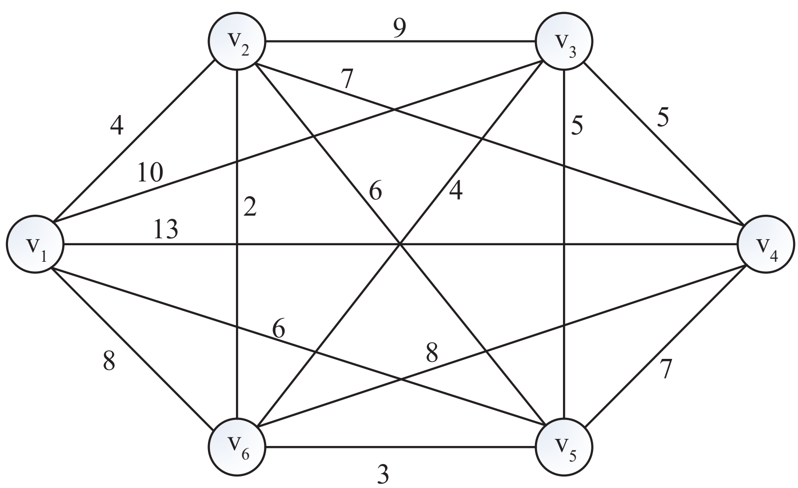

Figure 1. Initial graph of the traveling salesman task

Then we write the mathematical formulation of the traveling salesman problem in the following form:

$$\begin{gathered}100x_{11}+4x_{12}+10x_{13}+13x_{14}+6x_{15}+8x_{16}+\\+100x_{22}+4x_{21}+9x_{23}+7x_{24}+6x_{25}+2x_{26}+\\+100x_{33}+10x_{31}+9x_{32}+5x_{34}+5x_{35}+4x_{36}+\\+100x_{44}+13x_{41}+7x_{42}+5x_{43}+7x_{45}+8x_{46}+\\+100x_{55}+6x_{51}+6x_{52}+5x_{53}+7x_{54}+3x_{56}+\\+100x_{66}+8x_{61}+2x_{62}+4x_{63}+8x_{64}+3x_{65}\to\min_{x\in\Delta_\beta},\end{gathered}\tag{7}$$

where the set of admissible alternatives Δ_β is formed by the following system of inequality-type constraints:

$$\begin{cases}x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16}=1;\\x_{21}+x_{22}+x_{23}+x_{24}+x_{25}+x_{26}=1;\\x_{31}+x_{32}+x_{33}+x_{34}+x_{35}+x_{36}=1;\\x_{41}+x_{42}+x_{43}+x_{44}+x_{45}+x_{46}=1;\\x_{51}+x_{52}+x_{53}+x_{54}+x_{55}+x_{56}=1;\\x_{61}+x_{62}+x_{63}+x_{64}+x_{65}+x_{66}=1;\\x_{11}+x_{21}+x_{31}+x_{41}+x_{51}+x_{61}=1;\\x_{12}+x_{22}+x_{32}+x_{42}+x_{52}+x_{62}=1;\\x_{13}+x_{23}+x_{33}+x_{43}+x_{53}+x_{63}=1;\\x_{14}+x_{24}+x_{34}+x_{44}+x_{54}+x_{64}=1;\\x_{15}+x_{25}+x_{35}+x_{45}+x_{55}+x_{65}=1;\\x_{16}+x_{26}+x_{36}+x_{46}+x_{56}+x_{66}=1;\\u_2-u_3+6x_{23}\le 5;\\\dots\\u_6-u_5+6x_{65}\le 5;\\x_{11},x_{12},x_{13},x_{14},x_{15},x_{16},\dots,x_{61},x_{62},x_{63},x_{64},x_{65},x_{66}\in\{0,1\};\\u_1,u_2,u_3,u_4,u_5,u_6\in\boldsymbol{R}^{\mathbf{1}}.\end{cases}\tag{8}$$

It should be noted that in the mathematical formulation of the traveling salesman problem (7) and (8), the first 6 constraints correspond to constraints (2), the next 6 constraints correspond to constraints (3), and of the 20 constraints of type (4), only the first and last are given. It is assumed that the variables: $x_{ii}=0\left(\forall i\in\{1,2,\dots,6\}\right)$ and therefore are not included in the formulation of problem (7).

Let's solve this problem using the MS Excel program. To do this, create a new book and perform some preparatory actions:

– let's solve this problem using the MS Excel program. To do this, create a new book and perform some preparatory actions:

– we enter into the MS Excel table the weights of the arcs of the original graph (Fig. 1), which represent the values of the coefficients of the objective function (7);

– let's introduce the formula for the objective function (7);

– we introduce the values of the left side of the constraints of the first 12 formulas of system (8);

– we introduce formulas corresponding to restrictions of type (4), while, for the convenience of further calculations, we equate the elements x_{ii} to 0.

The appearance of the MS Excel worksheet with the initial data for solving the problem of drawing up an optimal plan (the traveling salesman problem) has the following appearance (Fig. 2).

	A	B	C	D	E	F	G	H
1		Objective function coefficients						Objective function value
2		100	4	10	13	6	8	=SUMPRODUCT(B2:G7;B9:G14)
3		4	100	9	7	6	2	
4		10	9	100	5	5	4	
5		13	7	5	100	7	8	
6		6	6	5	7	100	3	
7		8	2	4	8	3	100	
8	Variables: Xij	Xi1	Xi2	Xi3	Xi4	Xi5	Xi6	Limitations 1:
9	X1j							=SUM(B9:G9)
10	X2j							=SUM(B10:G10)
11	X3j							=SUM(B11:G11)
12	X4j							=SUM(B12:G12)
13	X5j							=SUM(B13:G13)
14	X6j							=SUM(B14:G14)
15	Limitations 2:	=SUM(B9:B14)	=SUM(C9:C14)	=SUM(D9:D14)	=SUM(E9:E14)	=SUM(F9:F14)	=SUM(G9:G14)	
16	Variables: Ui							
17				Meanings of limitations:				
18	u2-uj+6×2j		0	=$C$16-D16+6*D10	=$C$16-E16+6*E10	=$C$16-F16+6*F10	=$C$16-G16+6*G10	
19	u3-uj+6×3j		=$D$16-C16+6*C11	0	=$D$16-E16+6*E11	=$D$16-F16+6*F11	=$D$16-G16+6*G11	
20	u4-uj+6×4j		=$E$16-C16+6*C12	=$E$16-D16+6*D12	0	=$E$16-F16+6*F12	=$E$16-G16+6*G12	
21	u5-uj+6×5j		=$F$16-C16+6*C13	=$F$16-D16+6*D13	=$F$16-E16+6*E13	0	=$F$16-G16+6*G13	
22	u6-uj+6×6j		=$G$16-C16+6*C14	=$G$16-D16+6*D14	=$G$16-E16+6*E14	=$G$16-F16+6*F14	0	

Figure 2. Initial data for solving the problem of developing an optimal plan (traveling salesman problem)

To further solve the problem, you should call the master solver. After that, we introduce the necessary restrictions. The general view of the solver wizard parameter specification dialog box, with the entered parameters of the objective function, the type of final data, and the constraints, looks like this (Fig. 3).

After setting the constraints and the objective function, you can begin searching for a numerical solution by clicking the Solve button. The MS Excel program will perform the calculations. Afterwards we will receive a quantitative solution, which is shown in Fig. 4.

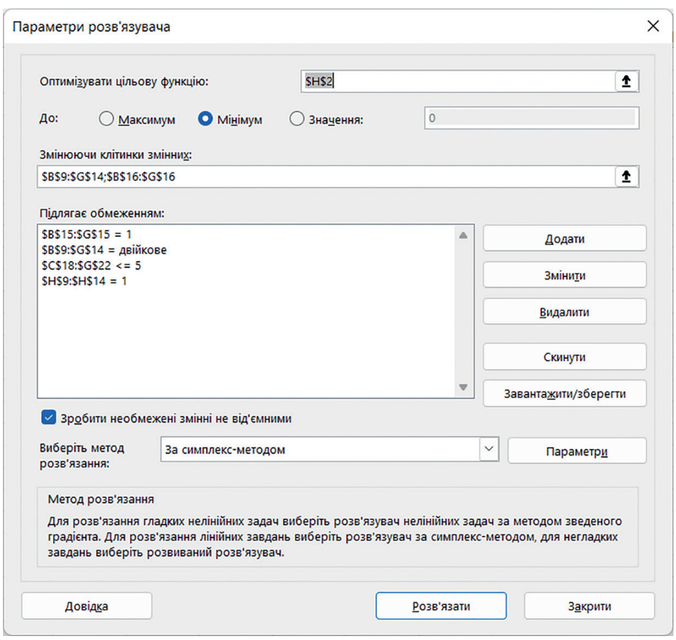

Figure 3. Limitation of variable values and parameters of the master solver for the task of developing an optimal transportation plan (the traveling salesman problem)

	A	B	C	D	E	F	G	H
1		Objective function coefficients						Objective function value
2		100	4	10	13	6	8	28
3		4	100	9	7	6	2	
4		10	9	100	5	5	4	
5		13	7	5	100	7	8	
6		6	6	5	7	100	3	
7		8	2	4	8	3	100	
8	Variables: *Xij*	*Xi1*	*Xi2*	*Xi3*	*Xi4*	*Xi5*	*Xi6*	Limitations 1:
9	*X1j*	0	1	0	0	0	0	1
10	*X2j*	0	0	0	0	0	1	1
11	*X3j*	0	0	0	1	0	0	1
12	*X4j*	0	0	0	0	1	0	1
13	*X5j*	1	0	0	0	0	0	1
14	*X6j*	0	0	1	0	0	0	1
15	Limitations 2:	1	1	1	1	1	1	
16	Variables: *Ui*	0	0	2	4	5	1	
17	Meanings of limitations:							
18	*u2-uj+6×2j*		0	-2	-4	-5	5	
19	*u3-uj+6×3j*		2	0	4	-3	1	
20	*u4-uj+6×4j*		4	2	0	5	3	
21	*u5-uj+6×5j*		5	3	1	0	4	
22	*u6-uj+6×6j*		1	5	-3	-4	0	

Figure 4. Result of quantitative solution of the problem of developing an optimal transportation plan (traveling salesman problem)

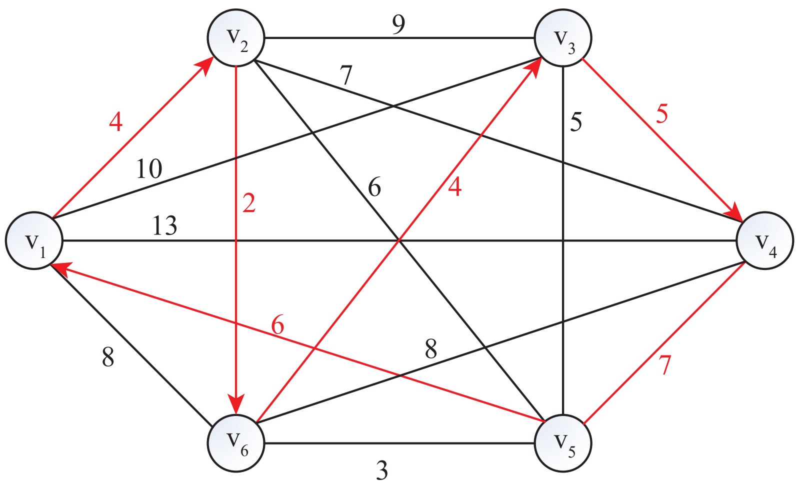

Figure 5. Complete closed path of minimum length in the original graph

The result of solving this problem of developing an optimal transportation plan (traveling salesman problem) are the found optimal values of the variables: $x_{12}=1$, $x_{26}=1$, $x_{34}=1$, $x_{45}=1$, $x_{51}=1$, $x_{63}=1$, the remaining variables are equal to 0. The found optimal solution corresponds to the value of the objective function: $f_{opt}=28$.

Analysis of the found solution shows that the closed route of minimum length, passing through all the vertices of the directed graph (Fig. 1), contains the following arcs: (1, 2), (2, 6), (6, 3), (3, 4), (4, 5), (5, 1). Thus, an optimal complete closed route was found, starting and ending at the vertex with number 1 and including a sequential visit to places in the city: from 1 to 2, from 2 to 6, from 6 to 3, from 3 to 4, from 4 to 5, from 5 to 1 (Fig. 5). The total length of this route will be minimal and equal to 28 *km.*

Conclusions. The article considered and analyzed the problem of logistics through solving the optimal planning problem, in particular the traveling salesman problem. The presented mathematical model allows us to formalize the process of finding the optimal route, which is relevant for a wide range of logistics tasks.

Using a practical example of delivering windows from a manufacturing plant to customers within the city, a route graph was constructed and calculations were implemented in MS Excel. The results showed the effectiveness of the proposed approach, as the optimal route minimized time and resource consumption.

The results obtained confirm the possibility of using mathematical models and digital tools to solve real logistics problems. Further research could be aimed at expanding the model to account for dynamic changes in logistics, such as congestion or changes in the number of delivery points, which would further increase the accuracy and flexibility of decision-making.

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