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# EXPERT ASSESSMENTS IN DECISION-MAKING PROBLEMS: THE METHOD OF MULTIPLICATIVE FUNCTIONS

# ЕКСПЕРТНІ ОЦІНКИ В ЗАДАЧАХ ПРИЙНЯТТЯ РІШЕНЬ: МЕТОД МУЛЬТИПЛІКАТИВНИХ ФУНКЦІЙ

*Expert judgments and fuzzy set theory are two different concepts of systems analysis, but their combination improves the mathematical tool of multi-criteria analysis of decision-making problems. The method of expert evaluations belongs to the class of partially formalized methods based on the use of intuition and experience of decision-makers. The multi-criteria nature of the decision-making problem is manifested in the search for an alternative that simultaneously satisfies the constraint and optimizes the vector quantity, the elements of which correspond to the objective functions of the problem. The choice of the optimal alternative, in most cases, is also ambiguous and is based on subjective, undefined information about the object of decision-making, or selection criteria that do not have quantitative assessments. In this case, it can be said that the decision-making process is fuzzy. Multiplicative Function Method is one of the approaches to modeling fuzzy systems, problems related to fuzzy inference, or fuzzy model description. The purpose of the work is to study the possibility of combining the theory of fuzzy sets with a deterministic algorithm for a multi-criteria decision-making problem.*

***Keywords:*** *algorithm, alternative, priority vector, fuzzy set, membership function, multiplicative function, synthesis.*

*Експертні оцінки та теорія нечітких множин є двома різними концепціями системного аналізу, але їх поєднання удосконалює математичний інструмент багатокритеріального аналізу задач прийняття рішення. У випадку, коли вибір рішення ґрунтується не на об'єктивних оцінках, а на судженнях менеджера чи споживача, говорять про експертні методи прийняття рішень. Метод експертних оцінок відноситься до класу частково формалізованих методів, що ґрунтуються на використанні інтуїції та досвіду осіб, які приймають рішення. Багатокритеріальність задачі прийняття рішення проявляється в пошуку альтернативи, що одночасно задовольняє обмеженням і оптимізує векторну величину, елементи якої відповідають цільовим функціям задачі. Ці функції утворюють математичний опис критерію якості альтернативи. Вибір оптимальної альтернативи, в більшості випадків, теж неоднозначний і ґрунтується на суб'єктивній, не визначеній інформації про об'єкт прийняття рішення, або критерії вибору, що не мають кількісних оцінок. У такому випадку можна говорити, що процес прийняття рішення відбувається в нечітких умовах. Використання нечітких оцінок альтернатив, отриманих на основі висновків експертів, є засобом врахування колективного знання та досвіду фахівців в умовах не структурованої суб'єктивності інформації, неможливості застосування детермінованих алгоритмів обґрунтування рішень. Метод мультиплікативних функцій (Multiplicative Function Method) є одним із підходів до моделювання нечітких систем, зокрема, при рішенні задач, що пов'язані з нечітким висновком, або нечітким описом моделі. Цей метод ґрунтується на операції множення нечітких функцій – функцій належності у просторі нечітких множин. Перевагами методу мультиплікативних функцій є простота обчислень, лінгвістична інтерпретація, можливість аналітично описати взаємодію між критерія-*

*ми та ваговими коефіцієнтами, визначеними на основі суджень експертів. Метою роботи є дослідження детермінованого алгоритму багатокритеріальної задачі прийняття рішення у поєднанні з засобами теорії нечітких множин.*

***Ключові слова:*** *алгоритм, альтернатива, вектор пріоритетів, нечітка множина, функція належності, мультиплікативна функція, синтез.*

**Formulation of the problem.** The main problem of modeling decision-making situations is the specification of uncertain characteristics of the management object and the quantitative assessment of descriptive criteria for choosing alternatives. The selection criteria, which comprise a combination of quantitative and qualitative assessments of alternatives within the task, may inherently describe conflicting interests of different decision-making parties. The choice of method for constructing and analyzing models of such problems depends on the degree of uncertainty in the decision-making problem situation. In the case when the choice of decision is not based on objective assessments, but on the judgments of the manager or consumer, we speak of expert decision-making methods. The method of expert assessments belongs to the class of partially informalized methods based on the use of intuition and experience of decision-makers. The legality of using expert assessments is based on assumptions: the undefined characteristic of the investigated phenomenon is a random variable, and the individual assessment of an expert specialist is interpreted as a separate implementation of this random variable. Accordingly, the mathematical expectation of the studied characteristic is within the range of group estimates of experts, that is the generalized collective judgment can be considered reliable. But, for a certain class of decision-making problems, this assumption is not true. Sometimes the opinion of an individual expert, regarding a little-researched problem, deserves more attention and may turn out to be more significant, and it will be lost during the formal processing of expert data. Therefore, most scientific developments of decision-making methods should activate the use of intuition and personal experience of experts.

The multi-criteria nature of the decision-making problem is manifested in the search for an alternative that simultaneously satisfies the constraint and optimizes the vector quantity, the elements of which correspond to the objective functions of the problem. These functions form a mathematical description of the alternative quality criterion. The choice of the optimal alternative, in most cases, is also ambiguous and is based on subjective, undefined information about the object of decision-making, or selection criteria that do not have quantitative assessments. In this case, it can be said that the decision-making process takes place in unclear conditions.

Expert judgments and fuzzy set theory are two different concepts of system analysis, but their combination will improve the mathematical tool of multi-criteria analysis of decision-making problems.

**Analysis of recent research and publications.** Decision-making is the process of choosing between different ways to achieve a goal. In the theory of system analysis, the key requirement for management models is formulated as follows: an adequate and reliable solution will be the result of a choice from a set of alternatives [1]. System analysis does not offer a universal algorithm for forming a set of alternatives and selection criteria. The system model provides organizations with a general framework and approaches to effective management of activities, adapting to the specific needs and conditions of each specific organization [2; 3]. The methodology of system analysis is a combination of various methods, tools, algorithms and fundamental concepts, united by one goal – building an adequate management system for the decision-making process. Informal methods of system analysis, the basis of which is the description of analytical procedures at the level of logic, subjective judgments of the decision-maker, are usually not associated with the use of mathematical apparatus. However, system analysis uses the mathematical apparatus not as a tool, but as an auxiliary tool of the decision-making process, and this application of mathematical algorithms is the most effective for non-typical economic problems [4]. The method of expert evaluations belongs to the class of partially informalized methods. In multi-criteria decision-making tasks, expert evaluations play a key role in determining the weighting coefficients of the evaluation criteria of alternatives. Many scientific works are devoted to the study of the features of the application of expert assessments in decision-making tasks [5-8]. They consider the forms of expert surveys, approaches to evaluating expert judgments, methods of processing survey results, requirements for experts and the formation of expert groups, issues of training experts, assessing their competence, methods of organizing expert surveys [7; 8]. Fuzzy set theory is a mathematical theory for modeling fuzzy or implicit concepts that cannot be uniquely defined or described using traditional logical sets [9; 13; 14]. This theory was first proposed by Lotfi Zadeh in 1965 [10-12]. It allows you to operate with vague variables and provides effective means of formalizing uncertainty of various nature, including linguistic forms of expert judgment, which more adequately reflect the qualitative characteristics of the real world [9]. The use of expert evaluations in combination with the theory of fuzzy sets allows solving complex decision-making problems in situations of vagueness or insufficient accuracy of input data. This can be particularly useful in areas where there is vagueness, ambiguity or uncertainty in determining the decision selection criteria.

**Formulation of the purpose of the article.** The purpose of the work is to combine the theory of fuzzy sets with a deterministic algorithm for a multi-criteria decision-making problem.

**Presentation of the main material.** The application of elements of the theory of fuzzy sets in the expert evaluation of alternatives allows to express fuzzy notions of the relation type, such as "better", "worse", "significantly prevails", "not significantly prevails", etc., with the help of fuzzy sets and fuzzy rules [14]. This allows to algorithmize models of situations in which exact numerical values are unavailable or uncertain. The method of multiplicative functions is one of the approaches to the modeling of fuzzy systems, in particular, when solving problems related to a fuzzy conclusion or a fuzzy model description. This method is based on the operation of multiplication of fuzzy functions – membership functions in the space of fuzzy sets.

In the general case, if we consider two fuzzy sets *A* and *B* with membership functions $\mu_A(X), \mu_B(X)$ to the universal set $X = \{x_i\}$, then the method of multiplicative functions determines the membership function for their product (intersection of fuzzy sets *A* and *B* at a point $x_i$), as the product of their membership functions at a point $x_i$.

$$\mu_{A\times B}(x_i) = \mu_A(x_i) \times \mu_B(x_i). \quad (1)$$

In multicriteria decision-making problems [15], the method of multiplicative functions can be used to evaluate alternatives. The basic idea is to calculate the product of the scores of each criterion for each alternative and then compare these products to determine the best alternative.

Consider a simplified decision-making model defined by a fuzzy set of criteria *K* with a membership function $\mu_K(A_i)$ and a fuzzy set of constraints *S* with a membership function $\mu_S(A_i)$ to a universal, limited set of alternatives $A = \{A_i\}, (i = 1 \div m)$. Due to the commutativity of the multiplication operation (1), the goals and constraints are symmetric with respect to the solution and there is no difference between them within the universal set of alternatives. Then, the solution to the problem of choosing an alternative will be a cross-section $\mu_{K\times S}(A_i)$ sets of vague goals and limitations

$$\mu_{K\times S}(A_i) = \mu_K(A_i) \times \mu_S(A_i). \quad (2)$$

The membership function, in the theory of fuzzy sets, usually takes values in the range [0;1], where 0 is no membership, 1 is full membership. The closer the value of the function $\mu_K(A_i)$ to one, the higher is the assessment of the significance of the alternative $A_i$ according to the criterionза критерієм $K = \{K_j\}$. The experts' task will be to organize the elements of a set of alternatives $A = \{A_1, A_2, \cdots, A_m\}$ by the criteria vector $K = \{K_1, K_2, \cdots, K_n\}$.

Let the numerical value $\mu_{K_j}(A_i)$ міститься в межах [0;1] is contained within [0;1] and determines the level of significance of the alternative $A_i$ according to the criterion $K_j$. Then, each criterion $K_j$ can be represented as a fuzzy set of ordered pairs of the universal set of alternatives $A$

$$K_j = \{A_i; \mu_{K_j}(A_i)\}; (j = 1 \div n; i = 1 \div m), \tag{3}$$

where $\mu_{K_j}(A_i)$ – the membership function of an alternative to a fuzzy set of criteria.

Evaluate the components $\mu_{K_j}(A_i)$ can be based on expert pairwise comparisons on the Thomas Saati scale [16, 17]. For this, symmetric matrices of pairwise comparisons of alternatives are constructed $A(K_j)$ for each criterion:

$$A(K_j) = (\alpha_{ik}) = \begin{pmatrix} \alpha_{11} & \cdots & \alpha_{1m} \\ \vdots & \ddots & \vdots \\ \alpha_{m1} & \cdots & \alpha_{mm} \end{pmatrix}, \tag{4}$$

where $\alpha_{ik}; (i = 1 \div m, k = 1 \div m)$ – numerical priority value $A_i$ over the alternative $A_k$ according to the criterion $K_j$.

The total number of pairwise comparison matrices is equal to the number of criteria [18]. If the experts' judgments are consistent, then the matrix of pairwise comparisons is inversely symmetric (if $\alpha_{ik} = \beta$, then $\alpha_{ki} = 1/\beta$); transitive ($\alpha_{is} \cdot \alpha_{sk} = \alpha_{ik}$). The consistency of the matrix means that the relative weights of the alternatives in the matrix of pairwise comparisons retain their value, regardless of how the comparison is performed. From a mathematical point of view, consistency is a linear dependence of the rows of the matrix (4).

Based on the matrix of pairwise comparisons, the component membership functions are calculated $\mu_{K_j}(A_i)$ – vectors of weighting coefficients of alternatives $\vec{V}^{A_i}$ according to a number of criteria

$$A(K_j) = \alpha \cdot \vec{V}, \tag{5}$$

where $\vec{V} = (v_1, v_2, \cdots v_i, \cdots v_m)$ – the principal eigenvector of the matrix $A(K_j)$; $\alpha$ – the maximum eigenvalue of the matrix $A(K_j)$.

If the main eigenvector of the matrix is normalized, then it can be considered a vector of local priorities of alternatives within the criterion $K_j$.

Vector of criteria priorities $\vec{V}^K$ can be determined from similar calculations of the matrix of pairwise comparisons, or by nonparametric methods, such as the rank correlation method, or as a vector of initial conditions that reflects the hierarchical construction of the solution.

After obtaining the priority vectors of the alternatives, fuzzy decision synthesis methods such as weighted sum or multiplicative function methods (2) can be applied to calculate the significance score of each alternative and select the optimal solution. There are several ways to construct multiplicative functions (multiplicative evaluation, integral index method, geometric mean method).

The algorithm of the multiplicative function synthesis method consists of the following steps.

Evaluate alternatives according to each criterion separately.

The main eigenvector of the criteria is calculated $\vec{V}^K$ and principal eigenvectors of the matrices of pairwise comparisons of alternatives $\vec{V}^{A_i}$;

Vector normalization is carried out by the geometric mean according to the criteria $\vec{V}^K$ and $\vec{V}^{A_i}$. For this, we find the geometric mean value of the elements for each row of the matrix of pairwise comparisons $\overline{\alpha_i}$ and perform rationing $v_i = \overline{\alpha_i} / \sum_{i=1}^{m} \overline{\alpha_i}$.

A matrix of weighting coefficients of alternatives is formed $(A_K)$:

$$(A_K) = (\vec{V}^{A_1} \quad \vec{V}^{A_2} \quad \cdots \vec{V}^{A_m}).$$

The elements of the membership function of a fuzzy set of decisions are calculated as the product of the priority matrix of alternatives by the vector of criteria

$$\mu_{K_j}\left(A_i\right)=\vec{V}^{K} *\left(A_K\right). \quad (5)$$

The highest numerical value of the function $\mu_{K_j}\left(A_i\right)$ indicates the best alternative.

**Conclusions.** The use of information received from experts in the development of a decision is closely related to the form of its presentation and processing methods. The entire set of data received from experts is a comparison of situations, objects, alternatives according to selected indicators, variables and parameters. At the same time, the comparison procedure is based on the existence of a principled possibility of establishing certain relations of order between the characteristics of decision-making processes and linguistic variables or elements of fuzzy sets used to represent qualitative evaluations of criteria and alternatives.

The advantages of the method of multiplicative functions are simplicity of calculations and linguistic interpretation. These methods of multiplicative functions allow to take into account the interaction between the criteria and weighting factors, which are established on the basis of expert judgments. However, they may require a large number of expert judgments, and their accuracy may depend on the correct definition of weighting factors. The problem is also the evaluation of the elements of fuzzy sets with a zero value of the membership function.

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