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ALGORITHM OF FUZZY PRIORITIES FOR ALTERNATIVE SOLUTIONS

АЛГОРИТМ НЕЧІТКИХ ПРІОРИТЕТІВ АЛЬТЕРНАТИВНИХ РІШЕНЬ

It is rather problematic to approximate the real decision management process with a mathematical model. Most decisions in the economy are complex in structure, multi-criteria tasks containing a set of descriptive, probabilistic, undefined characteristics. Such problems are solved by methods of system analysis. One of these methods is the method of analyzing hierarchies, which is used for models of personnel management, resource allocation, and evaluation of investment projects. The possibility of practical application of the method of analysis of hierarchies is limited by the requirement of complete certainty of alternative evaluations. Practical situations are characterized by uncertainty and limited information about the object or the decision-making process. Combining a systems approach with elements of fuzzy logic can correct this situation for the better. System models developed on the basis of the theory of fuzzy sets use the method of paired comparisons to rank management objects by factors and determine the importance of factors within the model. The article examines the algorithm for the synthesis of the vector of weighting coefficients of alternative decisions based on the matrix of pairwise comparisons with fuzzy estimates of alternatives. The possibility of combining elements of the theory of fuzzy sets and the method of analysis of hierarchies for the problem of decision-making is considered.

Key words: *alternative, criteria, vector of priorities, relation T. Saati, method of analysis of hierarchies, fuzzy set, decision making.*

Математичні моделі підтримки прийняття рішень достатньо ефективний апарат, якщо система, процес, або об'єкт прийняття рішення детерміновані за параметрами, змінними та структурними зв'язками. Реальний процес управління апроксимувати математичною моделлю досить проблематично. Більшість рішень в економіці є складними за структурою багатокритеріальними задачами, що містять множину описових, імовірнісних, або частково не визначених характеристик. Такі задачі розв'язуються методами системного аналізу. Одним з таких методів є метод аналізу ієрархій, який застосовується для моделей управління персоналом, розподілу ресурсів, оцінки інвестиційних проектів. Можливість практичного застосування методу аналізу ієрархій обмежена вимогою повної детермінованості точкових оцінок альтернатив. Практичні ситуації характеризуються обмеженістю інформації про об'єкт або процес прийняття рішення. Поєднання системного підходу з елементами нечіткої логіки може виправити цю ситуацію на краще. Системні моделі, розроблені на основі теорії нечітких множин, використовують метод парних порівнянь для ранжування об'єктів за факторами та визначення значимості факторів в межах моделі. Вектори локальних пріоритетів, визначені співвідношеннями Т. Сааті фактично описують функцію належності об'єкта до нечіткої множини характеристик, параметрів та критеріїв. У статті досліджується алгоритм синтезу вектора вагових коефіцієнтів альтернативних рішень на основі матриці парних порівнянь з нечіткими оцінками альтернатив. Розглянуто можливість поєднання елементів теорії нечітких множин та методу аналізу ієрархій для задачі прийняття рішень. Використано алгоритм співвідношень Т. Сааті для формування матриць парних порівнянь критеріїв задачі прийняття рішення. Наведено теоретичне обґрунтування можливості застосу-

вання з цією метою нечіткої логіки. Приведено приклад побудови симетричної матриці відображення критеріїв на множину альтернатив з послідуючим синтезом вектора вагових коефіцієнтів альтернативних рішень.

Ключові слова: *альтернатива, критерії, вектор пріоритетів, відношення Т. Сааті, метод аналізу ієрархій, нечітка множина, прийняття рішення.*

Formulation of the problem. The problem of developing decision-making support methods is quite urgent. Traditional deterministic methods and models of mathematical modeling of management decisions are not effective enough, and methods of mathematical statistics are not suitable for the analysis of qualitative data and not clearly defined characteristics of the management object. This significantly narrows the scope of using the mathematical apparatus for solving practical problems characterized by uncertainty and incomplete information about the objects or processes being studied.

An example of a mathematical approach to solving complex decision-making problems is the method of hierarchical composition of decision-making stages – the method of analysing hierarchies, or the decision tree method, which is often used to solve a wide range of tasks related to resource allocation, investment evaluation, personnel decision-making, etc. A classic decision tree is a way of representing a choice algorithm in a hierarchical decision structure. One of the disadvantages of this method is the possibility of processing only point estimates of decision-making stages, which are usually the probability distribution of algorithm directions at a certain level of the hierarchy [1, p. 526]. Point estimates of prior probability distributions are the result of statistical analysis, lengthy statistical studies, and cumbersome calculations. This significantly narrows the scope of application of the specified method for solving practical problems characterized by uncertainty and incomplete information about the studied objects or processes. The non-determinism of the input data and qualitative parameters of the decision-making model can be compensated by the use of fuzzy methods of determining the priority vector – the method of determining the function of the criterion belonging to a fuzzy set based on the matrix of pairwise comparisons. The method of paired comparisons is used to find indicators of the preference of objects according to various factors and indicators of the importance of the factors themselves [2; 3]. In the decision-making problem, the application of the method of paired comparisons actually determines the function of the belongingness of the alternatives to a fuzzy set of characteristics, parameters, and estimates at each decision-making step. The article examines the algorithm for the synthesis of weighting coefficients of alternative solutions based on the matrix of pairwise comparisons with fuzzy estimates.

Analysis of recent research and publications. Decision-making is a process of purposeful selection from a set of alternatives [4-6]. The consequence of the decision is the achievement of a certain result, which is evaluated according to the selected criteria. Decisions should be based on reliable current and forecasted information, analysis of all factors influencing decisions, take into account the possible consequences of choosing an alternative. Formally, the decision-making problem can be presented as a ratio of three sets $F\{A,S,K\}$: set of possible alternatives $A=\{A_l\}, l=1\div k$; set of states of nature $S=\{S_j\}, j=1\div m$ and a set of criteria for evaluating alternatives $K=\{K_i\}, i=1\div n$. The degree of certainty of the sets *A, S, K* determines the choice of modeling tools.

In situational decision-making problems, the case is quite common when information regarding the criteria for choosing alternatives is subjective in nature, contains parameters and structural relationships with significant uncertainty, which have no analogues in traditional mathematical language. In order to adapt the mathematical tool of analysis of decision-making systems to such problems, a new mathematical direction was created – the theory of fuzzy sets, which allows formalizing vague concepts and judgments that a person uses to describe his desires, goals, ideas about the surrounding world [3; 7]. Introducing the concept of a fuzzy set is an attempt to mathematically formalize fuzzy information.

This concept is based on the idea that the element belongs to a fuzzy set, based on the selected feature, not uniquely defined in the "yes-no" relationship. That is, a fuzzy set is formed from elements of different belonging ratios (from 0 to 1). According to this approach, belonging to the type $\left[x_i \in X = \{x_i\}, i = 1 \div n\right]$ loses its meaning, since it is necessary to indicate "to what extent" or "with what probability" a specific element x_i satisfies the properties of the set X.

The theory of fuzzy sets provides effective means of formalizing uncertainty of various nature, more adequately reflects the qualitative characteristics of the real world. Mathematical tools of fuzzy logic [8; 9] allow building an adequate model of reality. Elements of the theory of fuzzy sets are successfully applied in models of practical management problems, the characteristic feature of which is the lack of accuracy of information about the object of research, the non-determinism of criterion evaluations [10; 11]. A fuzzy model of decision-making better describes the process of human thinking than traditional logical systems [12]. One of the topical directions of the new theory has become the study of the fuzzy decision-making model.

Formulation of the purpose of the article. The purpose of the article is to study the algorithm for the synthesis of weighting coefficients of alternative solutions of a decision with fuzzy criteria.

Presentation of the main material. The complexity of the method of hierarchical analysis lies in the determination of weighting coefficients for evaluating alternative solutions. If the problem contains a bounded set $K = \{K_i\}$, then the matrix of pairwise comparisons $(K)_{i \times j}$, which reflects the judgment of the decision-maker, regarding the priorities of the criteria, will be symmetric, *i*-order (*i=j*). The criterion in the row of the matrix K is evaluated in pairs with each criterion of the column in integers from 1 to 9 according to the scale of T. Saati ratios. The comparability of such numerical equivalents ensures the fulfillment of the condition: if $k_{ij} = n$, then $k_{ji} = {}^{1}\!/_{n}$. The eigenvector of the matrix *K* determines the ordering of the relative evaluations of priorities. Given that pairwise comparison matrices are built on the basis of human judgment and assumptions, some degree of inconsistency between the elements of the *K* matrix is to be expected. Such inconsistency is not critical, if it does not go beyond the defined limits [1, pp. 519–521]. The consistency condition of the matrix of pairwise comparisons can be written as follows: $K \cdot \vec{V} = \alpha \cdot \vec{V}$, де $\vec{V}$ – vector of weighting coefficients, α – the maximum eigenvalue of the matrix *K*. Taking into account that the matrices of pairwise comparisons are inversely symmetric, the consistency of such matrices is determined by the maximum eigenvalue of the matrix, followed by a comparison of the obtained value with the dimension value – the order of the matrix ($\alpha = 1 \div n$). The closer the value of the maximum eigenvalue of the matrix is to the order of the matrix, the more consistent the elements of the matrix of pairwise comparisons are.

If the main eigenvector of the matrix is normalized, then it can be considered a vector of local priorities, or a vector of weighting coefficients of alternatives. Components v_i vector $\vec{V}$ can be calculated using an approximate normalization algorithm:

– for each row of the matrix K, we find the average value of the elements $\overline{k_i}$;

– we calculate the sum of the average values of the rows of the matrix $\sum_{i-1}^{n} \overline{k_i}$;

– we carry out rationing $v_i = \overline{k_i} \Big/ \sum_{i-1}^{n} \overline{k_i}$.

Vectors of weighting coefficients are determined for each element of the set $\{K_i\}$. Next, the synthesis procedure is performed - sequential determination of priority vectors of alternatives relative to criteria. The calculation of the priority vectors is performed taking into account the relationships between the criteria belonging to different levels of the hierarchical structure of the task.

Let's consider the procedure for building a vector of priorities on the example of analyzing the quality of the marketing strategy of a trading company selling the same type of product from three different manufacturers *A*, *B*, *C*. Suppose that as a result of marketing research, quality indicators of the product that determine the level of consumer demand are determined: *J* – aesthetics, *R* – reliability, *E* – ergonomics. The significance of indicators was determined according to T. Saati ratio scale. The *R* component is rated as significantly more important for quality assessment than *E* and does not significantly outweigh *J*. Then, the ratio of the hierarchy of criteria can be presented as a matrix of K – pairwise comparisons with the subsequent calculation of weighting coefficients

$$K = \begin{pmatrix} 1 & 3 & 5 \\ 1/3 & 1 & 2 \\ 1/5 & 1/2 & 1 \end{pmatrix}, \ \overline{v^K} = \begin{pmatrix} 0,648 \\ 0,230 \\ 0,122 \end{pmatrix}; \sum_{i=1}^{3} v_i^K = 1.$$

Weighting coefficients of alternatives *A*, *B*, *C* according to criteria *R, J, E*, represented by separate matrices of pairwise relations

$$R = \begin{pmatrix} 1 & 1/2 & 1/5 \\ 2 & 1 & 2/5 \\ 5 & 5/2 & 1 \end{pmatrix}; \ J = \begin{pmatrix} 1 & 3 & 2 \\ 1/3 & 1 & 2/3 \\ 1/2 & 3/2 & 1 \end{pmatrix}; \ E = \begin{pmatrix} 1 & 2 & 3 \\ 1/2 & 1 & 3 \\ 1/3 & 1/3 & 1 \end{pmatrix}$$

$$\overline{v^R} = \begin{pmatrix} 0,112 \\ 0,179 \\ 0,709 \end{pmatrix}; \overline{v^J} = \begin{pmatrix} 0,534 \\ 0,150 \\ 0,316 \end{pmatrix}; \ \overline{v^E} = \begin{pmatrix} 0,523 \\ 0,332 \\ 0,145 \end{pmatrix}; \sum_{i=1}^{3} v_i^R = 1; \sum_{i=1}^{3} v_i^J = 1; \ \sum_{i=1}^{3} v_i^E = 1.$$

The procedure for synthesis of vectors of weighting coefficients of alternatives relative to criteria:

$$\left(\overline{v^K}\right)^T * \left(A_K\right) = \begin{pmatrix} R & J & E \end{pmatrix} = (0,648 \quad 0,230 \quad 0,122) * \begin{pmatrix} 0,112 & 0,179 & 0,709 \\ 0,534 & 0,150 & 0,316 \\ 0,523 & 0,332 & 0,145 \end{pmatrix} =$$

$$= \begin{pmatrix} 0,259 & 0,191 & 0,550 \end{pmatrix}.$$

The general algorithm of hierarchical synthesis consists of the following steps:

– the main eigenvectors of the criteria of the higher level of the hierarchy are calculated ($\vec{V}^K$) and the principal eigenvectors of the alternative ratio matrices ($\vec{V}^A$);

– vector normalization is performed ($\vec{V}^K$) та ($\vec{V}^A$);

– a matrix of weighting coefficients of alternatives is formed $\left(A_K\right)$;

– the vector of criterion evaluations of alternatives is calculated as the product of the matrix of priorities of alternatives and the vector of criteria.

Conclusions. The possibility of combining the elements of the theory of fuzzy sets and the method of analysis of hierarchies to evaluate the alternatives of the decision-making problem is considered. A simplified algorithm for determining the weighting coefficients of alternatives according to the criteria of a descriptive – fuzzy presentation is presented. To determine the relative coefficients of the importance of alternatives and criteria, the T. Saati ratio scale was used. The main eigenvectors of the matrices of comparisons of alternatives and criteria are chosen as vectors of local priorities. The priority vectors are calculated taking into account the relationships between the criteria belonging to different levels of the hierarchical structure of the task. An example of the synthesis of weighting coefficients of alternatives is given. The legality of the combination of fuzzy set theory tools and system analysis models in the decision-making problem is substantiated.

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