МАТЕМАТИЧНІ МЕТОДИ, МОДЕЛІ ТА ІНФОРМАЦІЙНІ ТЕХНОЛОГІЇ В ЕКОНОМІЦІ

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ASSESSMENT OF MARKET EQUILIBRIUM OF BASIC DYNAMIC MODELS

ОЦІНКА РИНКОВОЇ РІВНОВАГИ ОСНОВНИХ ДИНАМІЧНИХ МОДЕЛЕЙ

The main models used to assess the market equilibrium are analyzed. Refinements of the mechanism of interaction between supply and demand are proposed, which are necessary for practical forecasting in the stock market. Elements of differential calculus are used for forecasting. The solution of the differential equation shows that the movement of the market cannot be described by a single equation. If demand changes, then this will entail a change in supply, which balances the market. The full cycle of market fluctuations, by analogy with the cobweb pattern, is divided into 4 quarters of the π/2 period. An assessment is made of the stability of the market equilibrium for each period. To do this, on each π/2 period, we write down and solve the differential equation. On the basis of research, the dependence of the change in the amplitude of market fluctuations during one half-period and the equation of market fluctuations relative to the equilibrium point was obtained.

Key words: *dynamic models, assessment of market equilibrium, demand function, supply function, market price stability.*

Оборот алгоритмічної торгівлі на великих фондових майданчиках сьогодні сягає, за деякими даними, 70%. При цьому вже йдеться не просто про те, щоб випередити конкурентів у здійсненні транзакції, а й зуміти передбачити рух ціни. Зробити це можна, наприклад, за допомогою математичної формули, що враховує приховану ліквідність ринку за цієї ліквідності заявок на купівлю та продаж. Виснаження черги заявок на купівлю або продаж може свідчити про швидкий рух ціни. В роботі аналізуються основні моделі, що використовуються для оцінки ринкової рівноваги. Пропонуються уточнення механізму взаємодії попиту та пропозиції, необхідні для практичного прогнозування на фондовому ринку. Для прогнозування використовуються елементи диференціального обчислення. Рух ринку, у стані зміщення щодо точки рівноваги, задається диференціальним рівнянням. Рішення якого свідчить, що рух ринку неможливо описати одним рівнянням. Якщо змінити попит, це спричинить зміну пропозиції, яка врівноважує ринок. Для прогнозу розвитку ринку оцінюється його поведінка після будь-якого обурення. Наприклад, несподіване зниження (підвищення) обсягу ціни на акції підприємств. Повний цикл коливання ринку за аналогією з павутиноподібною моделлю розбивається на 4 чверті періоду π/2. Проводиться оцінка стійкості ринкової рівноваги кожному періоді. Провівши дослідження по всім чвертям періоду коливань, отримано закон коливань ринку протягом одного періоду при різному поєднанні еластичності попи-

ту й пропозиції. Отримано залежність зміни амплітуди коливань ринку протягом одного напівперіоду та рівняння коливань ринку щодо точки рівноваги. Фундаментальний метод, що застосовується для прогнозування, ґрунтується дослідженні сукупності різноманітних умов і факторів, що є на ринку, та розробки на основі економетричних моделей. При застосуванні технічного аналізу, на відміну від фундаментального, досліджується динаміка цін на торгах. З'ясовується їх тенденцій у минулому та переносяться виявлені закономірності на майбутнє. Проте вітчизняні реалії біржової торгівлі, які виявляються у нерозвиненості строкового біржового ринку, незначному обсязі та нерегулярності укладених угод, значно утруднюють застосування даних методів.

Ключові слова: *динамічні моделі, оцінка ринкової рівноваги, функція попиту, функція пропозиції, стійкість ринкової ціни.*

Formulation of the problem. Nowadays, to assess market equilibrium, two main dynamic models are used: the cobweb model and the Evans model. The web-like model quite clearly shows the process of interaction between supply and demand in markets, but their equations do not contain time as one of the main characteristics of the dynamic process (which includes all economic processes). Evans' model does not have this drawback. It makes an attempt to take into account the time factor.

Analysis of recent research and publications. The hypothesis about the existence of general economic equilibrium was originally set out in the work of L. Walras «Elements of Pure Political Economy». Pure political economy was conceived by Walras as a theory of price determination under a hypothetical regime of free competition. The author argued that «...this theory is mathematical, which means that although it can be stated in ordinary language, its proof must be mathematical. It is entirely based on the theory of exchange, which, in turn, is entirely expressed – in a state of market equilibrium – in a double fact (the central idea of the theory of general equilibrium): firstly, the fact that each participant in the exchange receives maximum utility, secondly, in the fact of equality of the volumes of demand and supply for each product for all participants. Only mathematics can give us the condition for maximum utility" [1]. These two facts underlie the equilibrium state in markets, according to Walras L. If the first fact defines equilibrium as the state of each individual economic entity, then the second fact fixes equilibrium as the state of the system of interacting entities as a whole. As new research appears, theoretical explanations (algorithms) of general economic equilibrium inevitably become more complex, incorporating approaches from related disciplines: game theory, algebra and linear programming. The theory of general economic equilibrium is becoming the leading approach for modeling socio-economic processes. Mathematical substantiation of the Walras hypothesis was carried out in the 1950s in the works of Arrow-Debre [2], McKenzie, Gale, Nikaido. Further work was carried out on the improvement of models and their generalization [3; 4]. The Evans model is a model in which an attempt is made to take into account the time factor [5; 6]. General economic equilibrium models are actively used as an applied tool in analyzing the effectiveness of economic policies, as well as the influence of various kinds of exogenous influences and endogenous changes on the economy at various levels.

Formulation of the purpose of the article. Study of the impact on the the volume of purchases that provide the seller with the greatest profit.

Presentation of the main material. For practical forecasting, it is necessary to clarify the mechanism of interaction between supply and demand in the stock market. Let us also apply elements of differential calculus for this. Let us depict the position of the market in a displaced state relative to the equilibrium point by a distance x (Figure 1) and assume that the market has balanced due to a corresponding increase in price by the value ΔP. In this case, the restoring force of the market will be equal to $-cx$, where c is the price elasticity (rigidity) of demand or supply.

The differential equation of market movement in this case has the form: $x'' = -cx$, or:

$$x'' + k^2 x = 0 \quad (1)$$

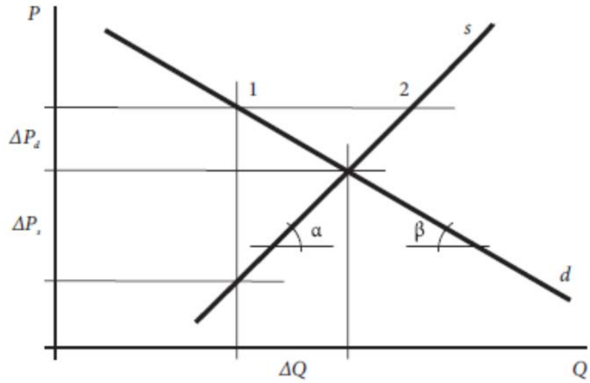

Figure 1. Elasticity (rigidity) of supply and demand

Where $k^2 = c$. Thus, choosing the origin of coordinates in the position of the static equilibrium of the market, we always obtain a differential equation of free fluctuations of the market without a constant term on the right side. The resulting differential equation (1) of free market fluctuations is a second-order linear homogeneous equation with constant coefficients. Its chara cteristic equation has the form: $\lambda^2 + k^2 = 0$ .The roots of the characteristic equation are imaginary: $\lambda_{1,2} = \pm ki$. Therefore, the solution of the equation is written in the form:

$$x = C_1 coskt + C_2 \sin kt. \tag{2}$$

To determine the integration constants C_1 and C_2, we differentiate solution (2):

$$x'' = -C_1 t \sin kt + C_2 t \cos kt. \tag{3}$$

Let us substitute into (2) the initial conditions $t = 0, x = x_0 = -\frac{\Delta P}{c}$,and in (3) $t = 0, x' = x'_0 = 0.$ We define integration constants: $C_1 = x_0 = -\frac{\Delta P}{c}, C_2 = 0.$ Equation (2) of market movements (market changes) after substituting the values of C_1 and C_2 will take the form $x = -\frac{\Delta P}{c} \cos kx,$ where $k = \sqrt{c}.$

At the same time, it is obvious that the market movement, in our case, cannot be described by a single equation. Indeed, when one of the main parameters of the market (for example, demand) changes, in accordance with general economic laws, a change in supply inevitably occurs, which balances the market. In this case, the elasticity (rigidity) of supply takes place, i.e., at point 2 (Figure 1), the changed state of the market is balanced due to the action of the market mechanism according to the neoclassical theory of market equilibrium. However, the elasticities of supply and demand are different, which is due to the initially opposite intentions of market participants (sellers and buyers). The difference in elasticity leads to a discrepancy in changes in the volumes of supply and demand. Therefore, for further analysis of the interaction between supply and demand, we introduce the designations c_s and c_d . In a formalized form, the dependences c_d and c_s have the form (Figure 1):

$c_d = \frac{\Delta P_d}{\Delta Q} = tg\beta$; $c_s = \frac{\Delta P_s}{\Delta Q} = tg\alpha$, where ΔQ - increment (change) in the volume of goods (shares, etc.) on the market; ΔP_d - increment (change) in price when the volume of supply of goods (shares, etc.) changes on the market equal to ΔQ ; ΔP_s - increment (change) in price with a change in the volume of demand for goods (shares, etc.) in the market equal to ΔQ . In other words, the larger the value of *c*, the less elastic supply or demand. To predict the development of the market, it is necessary to evaluate its behavior after receiving some kind of disturbance. Having divided the cycle (period equal to 2π) of market fluctuations by anal-

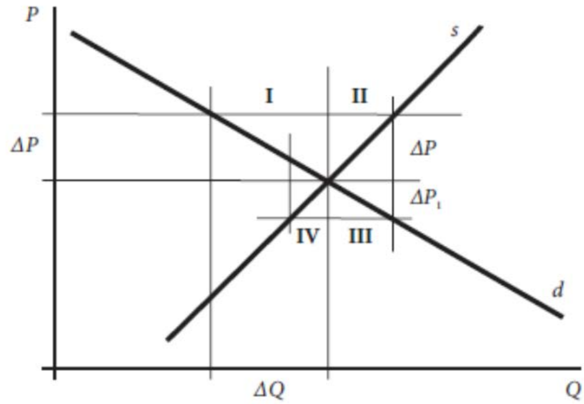

Figure 2. Scheme of market fluctuations (one period)

ogy with the cobweb model into 4 quarters of the period π/2, we will assess the stability of the market equilibrium as the ability to withstand these disturbances (Figure 2).

In the first quarter of the oscillation period (Figure 2) of market equilibrium, the differential equation has the form (1) or:

$$\frac{d^2Q}{dt^2} + k^2Q = 0, \text{ where } k^2 = c_d \tag{4}$$

Thus, choosing the origin of coordinates in the position of market equilibrium, we obtain a differential equation of free fluctuations without a constant term on the right side. The resulting differential equation (1) and (4) is a second-order linear homogeneous equation with constant coefficients. Its characteristic equation has the form

$$\lambda^2 + k^2 = 0 \tag{5}$$

The roots of the characteristic equation are imaginary: $\lambda_{1,2} = \pm ki$. Therefore, the solution of the equation is written in the form:

$$Q = C_1 coskt + C_2 \sin kt. \tag{6}$$

To determine the integration constants C_1 and C_2, we calculate

$$\frac{dQ}{dt} = -C_1 k \sin kt + C_2 k \cos kt. \tag{7}$$

Substituting into (6) the initial conditions $t = 0, Q = Q_0 = -\frac{\Delta P}{c_d}$, and in (7) $t = 0, \frac{dQ}{dt} = \frac{dQ_0}{dt} = 0,$ we get $C_1 = -\frac{\Delta P}{c_d}, C_2 = 0.$

After substituting the integration constants, we obtain the equation for market fluctuations in the first quarter of the period (Figure 3):

$$Q = -\frac{\Delta P}{c_d} \cos kt, \text{ where } k = \sqrt{c_d} . \tag{8}$$

Respectively

$$\frac{dQ}{dt} = \frac{\Delta P}{c_в} k \sin kt, \text{ where } k = \sqrt{c_d} . \tag{9}$$

Having carried out similar reasoning for the remaining three-quarters of the period of fluctuations, we come to the conclusion that, by successively solving the obtained equations of market fluctuations, we will obtain the law of market fluctuations during one period with different combinations of supply and demand elasticity (Figure 3, 4 and 5).

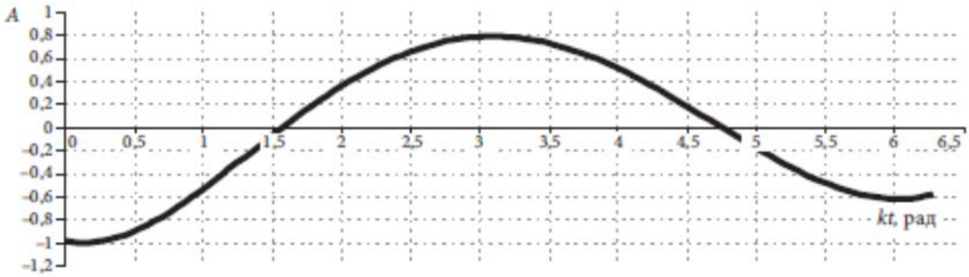

Figure 3. Period of market fluctuations for the case $c_d < c_s$ – supply is less elastic than demand (supply is more rigid than demand)

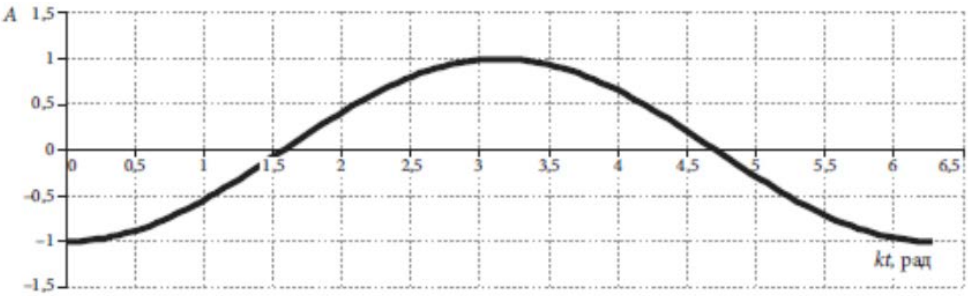

Figure 4. Period of market fluctuations for the case $c_d = c_s$ – supply and demand have the same elasticity

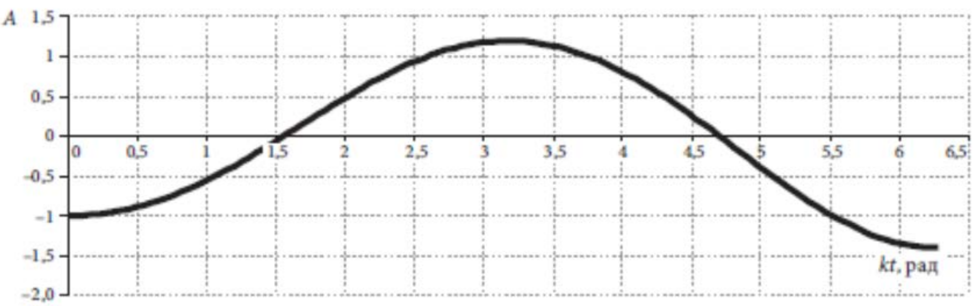

Figure 5. Period of market fluctuations for the case $c_d > c_s$ – supply is more elastic than demand (supply is less rigid than demand)

Carrying out similar reasoning, one can write the equations for price fluctuations (Figure 6).

Summarizing the above, we can derive the following dependence of the change in the amplitude of market fluctuations during one half-period:

$$Q_1 = Q_0 \frac{c_d}{c_s}; \; Q_2 = Q_1 \frac{c_d}{c_s} = Q_0 \left(\frac{c_d}{c_s} \right)^2; \; \dots \; Q_n = Q_0 \left(\frac{c_d}{c_s} \right)^n, \tag{10}$$

likewise

$$P_1 = P_0 \frac{c_d}{c_s}; \; P_2 = P_1 \frac{c_d}{c_s} = P_0 \left(\frac{c_d}{c_s} \right)^2; \; \dots \; P_n = P_0 \left(\frac{c_d}{c_s} \right)^n, \tag{11}$$

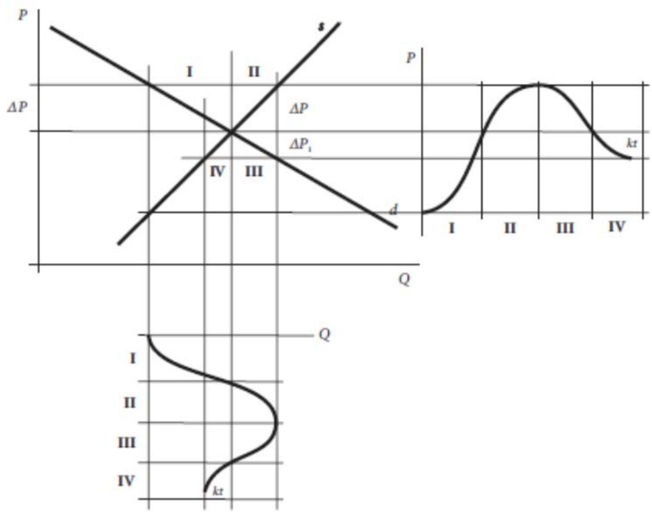

Figure 6. Graphical interpretation of fluctuations in the volume of goods (shares) and prices

where n is the serial number of the half-period of fluctuations of the market parameter; Q_0 and P_0 are the initial value of the deviation of the market parameter, in particular, volume and price, respectively. Thus, we write the solution of the equation of market fluctuations with respect to the equilibrium point

$$Q = Q_0 \left(\frac{c_d}{c_s} \right)^n \cos\left(kt + \phi_n\right) \tag{12}$$

or

$$P = P_0 \left(\frac{c_d}{c_s} \right)^n \cos\left(kt + \phi_n\right) \tag{13}$$

where ϕ_n is the initial phase of the nth half-cycle; k is the average frequency of free market fluctuations: $k = \sqrt{\frac{c_d + c_s}{2}}$. The full period of market fluctuations is determined by the formula: $T = \frac{2\pi}{k}$.

Analysis of equations (13) and (14) shows that the market will be in a state of stable equilibrium only when $c_d < c_s$ .That is, the elasticity of demand is greater than the elasticity of supply, or, in other words, the supply is more rigid in comparison with demand. Moreover, the magnitude of the largest deviations with their decrease changes the more intensively, the greater the rigidity of supply in relation to demand (Figure 7).

Thus, for $m > 1$, the equilibrium is unstable; for $m = 1$, the equilibrium is not defined; for $m < 1$, the equilibrium is stable (Figure 8).

Analysis of Figure 8 shows that with the same deviation from 1 for $m > 1$, the intensity of the increase in deviations is much greater than their decrease for $m < 1$. An analysis of equation $T = \frac{2\pi}{k}$ shows that the period of fluctuations will be the longer (longer), the greater the elasticity of supply and demand (less rigidity).

Conclusions. The presented free fluctuations of the market in their pure form (without noise) are impossible, since even during one period (especially since we have adopted a conditional period measured in radians), additional market disturbances can be observed, which will be superimposed on free fluctuations, significantly distorting the picture.

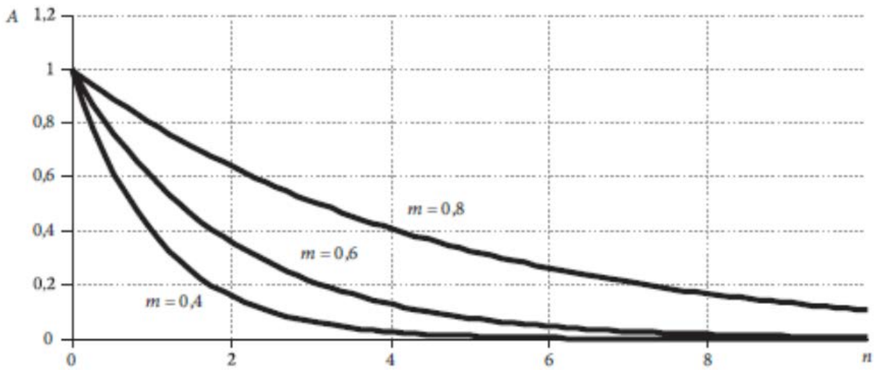

Figure 7. Intensity of change in the magnitude of the largest market deviations at $A_0 = 1$ *and* $m < 1$*, where* $m = \frac{c_d}{c_s}$

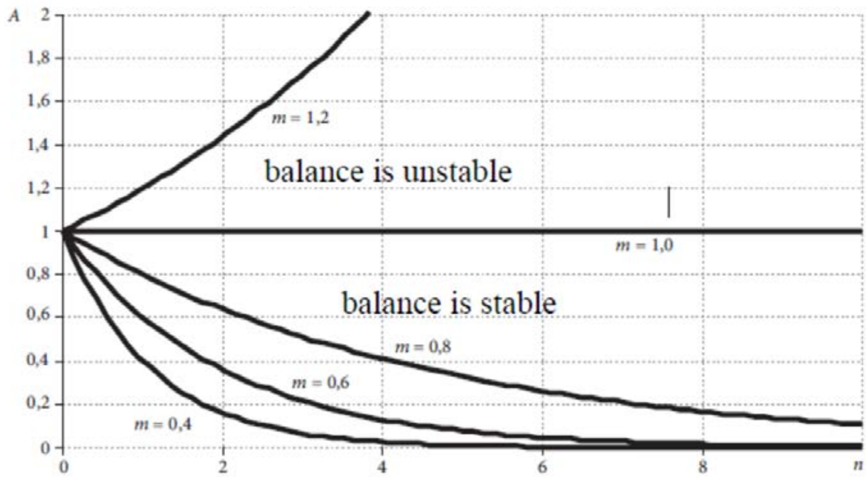

Figure 8. Intensity of change in the magnitude of the largest market deviations at $A_0 = 1$ *at* $m > 1$ *and* $m < 1$*, where* $m = \frac{c_d}{c_s}$

References:

1. Walras L. (1874) Elements d'Economie Politique Pure. *Revue de Théologie et de Philosophie et Compte-rendu des Principales Publications Scientifiques,* vol. 7, pp. 628–632. Available at: https://www.jstor.org/stable/44346456?seq=1#metadata_info_tab_contents

2. Arrow K. J., Debreu G. (1954) Existence of an equilibrium for a competitive economy. *Econometrica*, vol. 22, no. 3, pp. 265–290.

3. Kozak Yu. H. Matskul V. M. (2017) *Matematychni metody ta modeli dlia mahistriv z ekonomiky. Praktychni zastosuvannia: Navch. posib*. [Mathematical Methods and Models for Masters in Economics. Practical Applications: a textbook]. Kyiv: Tsentr uchbovoi literatury.

4. Debela I. M. (2011) *Ekonomiko-matematychne modeliuvannia: navchalnyi posibnyk.* [Economic and Mathematical Modeling: a textbook]. Kherson: Khersonska miska drukarnia. (in Ukrainian)

5. Bilousova T. P. (2021) Matematychna model optymalnoho rynku. [Mathematical model of the optimal market]. *Taurian Scientific Bulletin. Series: Economics*, vol. 8, pp. 70–75.

6. Bilousova T. P. (2023) Equilibrium price on the market of one good. evans model. *Taurian Scientific Bulletin. Series: Economics*, vol. 16, pp. 9–14.