ЕКОНОМІЧНА ТЕОРІЯ ТА ІСТОРІЯ ЕКОНОМІЧНОЇ ДУМКИ

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EQUILIBRIUM PRICE ON THE MARKET OF ONE GOOD. EVANS MODEL

РІВНОВАЖНА ЦІНА НА РИНКУ ОДНОГО ТОВАРУ. МОДЕЛЬ ЕВАНСА

In economic theory, the concept of equilibrium is important. This is the state of the object, which it retains in the absence of external influences. Achieving a balance between supply and demand is one of the main indicators of the effectiveness of the functioning of the country's economy in market conditions. There are many models for establishing an equilibrium price in the market for one product. The most famous equilibrium models are considered: L. Walras, A. Marshall, "spider" model with discrete time and Evans' model with continuous time. Evans's economic model for studying the establishment of an equilibrium price in the market of one product is considered. Its solution is given using the apparatus of differential equations. Graphs of the dependence of price on time are constructed, proving the main assumption of the model that the price changes depending on the relationship between supply and demand and its increase is directly proportional to the excess of demand over supply and the duration of this excess.

Key words: *equilibrium, economic model, demand, supply, differential equation.*

В економічній теорії важливим є поняття рівноваги. Це стан об'єкта, який він зберігає за відсутності зовнішніх впливів. Досягнення рівноваги між попитом і пропозицією є одним із основних показників ефективності функціонування економіки країни в умовах ринку. Ринкова рівновага – ситуація на ринку, при якій немає тенденції до зміни ринкової ціни або обсягу благ, що продаються. Рівноважна ринкова ціна – ціна, за якої величини попиту та пропозиції товару збігаються. Ринкова рівновага ціни та обсяг блага, що продається, можуть змінюватися у відповідь на зміни попиту та пропозиції. Існує багато моделей встановлення рівноважної ціни ринку одного товару. Розглядаються найвідоміші моделі рівноваги: Вальраса Л., Маршалла А., «павутиноподібна» модель з дискретним часом і модель Еванса з безперервним часом. Головним у підході Вальраса Л. відзначається різниця в обсягах попиту та пропозиції. Головним у підході Маршалла А. є різниця цін P_1 та P_2. Маршалл виходить із того, що продавці, перш за все, реагують на різницю ціни попиту та ціни пропозиції. Павутиноподібна модель – модель, що зображує траєкторію руху до стану рівноваги, коли реакція пропозиції чи попиту запізнюється. Вона визначає динамічний процес: траєкторію коригування цін та обсягу виробництва під час руху від одного стану рівноваги до іншого; використовується для опису коливань цін на ринках сільськогосподарської продукції; на біржовому ринку, де

пропозиція реагує зміну цін із деяким запізненням. Модель Еванса це модель з безперервним часом. Розглянуто економічну модель Еванса з вивчення встановлення рівноважної ціни на ринку одного товару. Наведено її розв'язання за допомогою апарату диференціальних рівнянь, та побудовані графіки залежності ціни від часу, що доводять основне припущення моделі, що ціна змінюється в залежності від співвідношень між попитом та пропозицією, її збільшення прямо пропорційно до перевищення попиту над пропозицією та тривалості цього перевищення. Також наведено деякі відомості про етапи побудови математичних моделей та історичні відомості, які допомагають простежити еволюцію економічного моделювання.

Ключові слова: *рівноважна ціна, економічна модель, попит, пропозиція, диференціальне рівняння.*

Formulation of the problem. Currently, two main dynamic models are used to assess market equilibrium: the "spider" model and the Evans model. The cobweb model quite clearly illustrates the process of interaction between supply and demand in organized markets, however, in the equations of supply and demand, there is no time as one of the main characteristics of dynamic processes (which include all economic processes). Evans' model does not have this shortcoming, in which an attempt is made to take into account the time factor. The characteristics of supply and demand determine the nature of market price stability. In this regard, the Evans model also requires clarification.

Analysis of recent research and publications. At the end of the 19th century, the equilibrium models of the economic system by A. Marshall, L. Walras, Arrow-Debré, Evans, the cobweb model, etc., appeared [1–3]. These models are considered classical models of economic equilibrium and assume the achievement of an equilibrium state of the economic system with the equality of aggregate demand and aggregate supply. The global financial and economic crisis (2008–2009) confirmed the fact that the real economic system develops cyclically and cannot be in equilibrium for a long time (no more than 50–60 years). For many years, work has been carried out to improve existing models and generalize them [3–6]. Evans' model with continuous time is interesting to study.

Formulation of the purpose of the article. The purpose of the work is to consider the economic model of Evans and to study the establishment of an equilibrium price in the market for one product. Solve it using the apparatus of differential equations, build graphs of price versus time. Show the basic assumption of the model that the price changes depending on the relationship between supply and demand. And that its increase is directly proportional to the excess of demand over supply and the duration of this excess.

Presentation of the main research material. Determination of the equilibrium price. When the price ceiling is set below the equilibrium price, there is a shortage and demand exceeds supply. This situation will lead to competition between buyers for the opportunity to buy this good. Competing buyers begin to offer higher prices. In response, sellers begin to raise prices. As prices rise, demand decreases and supply increases. This continues until the price reaches its equilibrium level (P; Q). When price floors are set above the equilibrium price, supply exceeds demand and there is a surplus of goods. Market equilibrium and deviations from it are shown in Figure 1.

There are four options for the influence of shifts in supply and demand curves on the price and volume of goods:

1. An increase in demand for a good causes a shift in the demand curve to the right, resulting in an increase in both the equilibrium price and the equilibrium quantity of the good.

2. A decrease in demand for a good shifts the demand curve to the left, resulting in a decrease in the equilibrium price and equilibrium quantity of the good.

3. An increase in the supply of a good shifts the supply curve to the right, resulting in a decrease in the equilibrium price and an increase in the equilibrium quantity of the good.

4. A decrease in the supply of a good shifts the supply curve to the left, resulting in an increase in the equilibrium price and a decrease in the equilibrium quantity of the good.

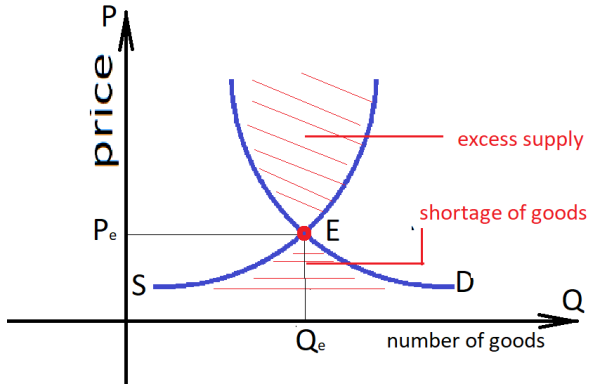

Figure 1. Market equilibrium and deviations from it
Market equilibrium. P_e – equilibrium price; Q_e – equilibrium quantity of a good

Using these four options for changing supply and demand and shifting their curves, it is possible to determine the equilibrium point in case of any fluctuations in supply and demand. However, the above-mentioned "four rules" of supply and demand do not always "work", because often, both the demand curve and the supply curve shift simultaneously, which greatly complicates the analysis of real economic phenomena and processes. A scientifically based method of supply and demand analysis involves supply:

1) distinguish between a change in demand or supply that leads to a shift in the curve, and a change in the amount of demand or supply that causes movement along the curve;

2) to keep all other conditions equal, which requires knowledge of the difference between the influence due to a change in the price of a good, and the influence due to a change in other factors.

The equilibrium price is one of the mechanisms for establishing market equilibrium. The equilibrium price is the price at which the volume of demand is equal to the volume of supply, in other words, this is the only price that meets the condition: $P_e = P_D = P_S$. At a given price, the equilibrium quantity of goods offered on the market is also established: $Q_e = Q_D = Q_S$. The equilibrium price performs the most important functions:

– informational – its value serves as a guideline for all market entities;

– normalizing – it normalizes the distribution of goods, giving a signal to the consumer about whether a given product is available to him and how much supply of a product he can count on at a given level of income. At the same time, it affects the producer, showing whether he can recoup his costs or whether he should refrain from production. Thus, the producer's demand for resources is normalized;

– stimulating – it forces the manufacturer to expand or reduce production, change technology and assortment so that the costs "fit" into the price and there is still some profit left.

The most famous equilibrium models: L. Walras, A. Marshall, "spider" model with discrete time and Evans model with continuous time.

The market of one product is considered, time is considered continuous. Let $d(t)$, $s(t)$, $p(t)$ be the demand, supply and price of this commodity at time t, respectively. Both demand and supply are considered to be linear functions of price, i.e. $d(p) = a - bp$, $a,b > 0$ – demand falls as price rises, $s(p) = \alpha + \beta p$, $\alpha,\beta > 0$ – supply increases as price increases. It is natural to assume that $a > \alpha$, that is, at a zero price, demand exceeds opposition (in other words, the product is desired). The basic assumption is that the price changes depending on the relationship between supply and demand: $dp = r(d - s)dt$. Where $r > 0$, that is, the price increase is directly proportional to the excess of demand over supply and the duration of this excess. So, we get the differential equation: $\frac{dp}{dt} = r(d - s)$.

Substituting into this equation the linear dependences of supply and demand on the price, we obtain a linear non-homogeneous differential equation with the initial condition:

$$\frac{dp}{dt} = -r\left((b+\beta)\cdot p - a + \alpha\right);\ p(0) = p_0.$$

This equation has a stationary point: $p^* = \frac{(a-\alpha)}{(b+\beta)} > 0$. It's clear that $\frac{dp}{dt} > 0$, if $p^* > 0$ and $\frac{dp}{dt} < 0$, if $p^* < 0$. The price p^* itself is the equilibrium price – when supply and demand are equal. The equilibrium price can also be found graphically – as a point of intersection of the supply and demand lines (Figure 1). But the assertion that equilibrium implies equality of supply and demand is incorrect. Indeed, supply and demand are functions, and the equality of functions implies a coincidence, not an intersection of graphs. Consequently, at the equilibrium point, the values (volumes) of supply and demand coincide, and not the graphs of functions.

Practical application of the Evans equilibrium price model. Let us describe the process of establishing an equilibrium price if time is continuous and the market for one commodity is considered. Demand D and supply S linearly depend on the price: and the price change is proportional to the excess of demand over supply with a proportionality coefficient.

$$D = 28 - 2p;\ S = 19 + p;\ \gamma = 1;\ p^* < p(0).$$

Let's consider different cases. Let's build graphs. On the basis of the resulting calculations and the constructed graphs, we will draw the appropriate conclusions.

The price increase is directly proportional to the excess of demand over supply with a proportionality coefficient. The Evans differential equation has the form: $\frac{dp}{dt} = \gamma(D - S)$. Substitute in this differential equation the linear dependence of supply and demand on the price, we obtain a differential equation: $\frac{dp}{dt} = -\gamma\left((b+\beta)\cdot p - a + \alpha\right)$. In our case: $\frac{dp}{dt} = -1\left((2+1)\cdot p - 28 + 19\right) = 9 - 3p$. The general solution to this equation would be: $p(t) = 3 - C\cdot e^{-3t}$. Let's find a point of stable equilibrium: $p^* = \frac{(a-\alpha)}{(b+\beta)} = \frac{28-19}{2+1} = 3$. The graph (Figure 2) shows that the integral curves given by the equation $p(t) = 3 - C\cdot e^{-3t}$ striving for a balance $p^* = 3$.

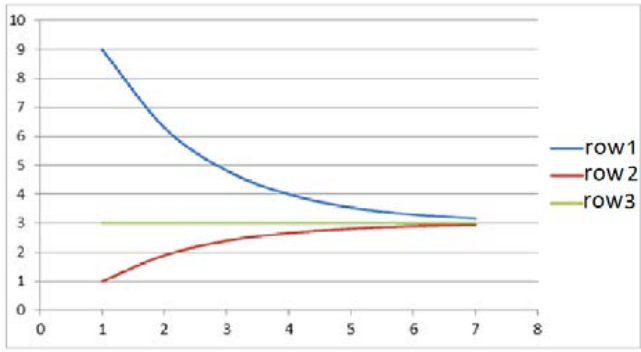

Figure 2. Integral curves given by the equation

At the initial moment of time: $p(0) = 9, p(0) > p^*, p^* - p(0) = -6, p(t) = 3 + 6\cdot e^{-3t}$. In this case, the price of the good decreases, approaching the equilibrium price. (row 1, Figure 2). If at the initial time: $p(0) = 1, p(0) < p^*$, then $C = 2$ and $p(t) = 3 - 2\cdot e^{-3t}$. In this case, the price of the good increases, approaching the equilibrium price (row 2, Figure 2). The stationary solution is stable, and deviation from it eventually leads to a return to the original state.

Similarly, consider and depict graphically cases of excess demand over supply and equality of demand to supply, $\beta > b$. Demand D and supply S are linearly dependent on price: $\beta = b$, $D = 8 - 0{,}5p$, $S = 2 + 1{,}5p$, $\gamma = \frac{1}{2}$ – proportionality factor. We get the differential equation: $\frac{dp}{dt} = 3 - p$. General solution of the differential equation in the form: $p(t) = 3 - C \cdot e^{-t}$, $p^* = \frac{a - \alpha}{b + \beta} = \frac{8 - 2}{0{,}5 + 1{,}5} = 3$, $p(0) = 4$, $p(0) > p^*$. Then C = -1, respectively $p(t) = 3 + 1 \cdot e^{-3t}$.

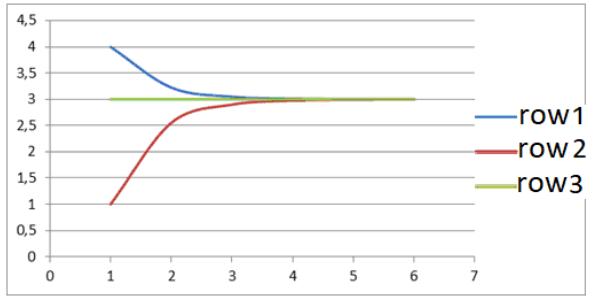

Figure 3. The price of a commodity decreases, approaching the equilibrium price

If $p(0) = 1, p(0) < p^*$, then $C = 2 \Rightarrow p(t) = 3 - 2 \cdot e^{-3t}$. The price of a commodity increases, approaching the equilibrium price (row 2, Figure 3).

Let the linear supply and demand equations be as follows: $D = 28 - 3p$, $S = 19 + 3p$. We compose the Evans differential equation: $\frac{dp}{dt} = 3(3 - 3p)$. General solution of the equation describing the dynamics of the equilibrium price: $p(t) = 1{,}5 - Ce^{-6t}$; $\square^* = 1{,}5$.

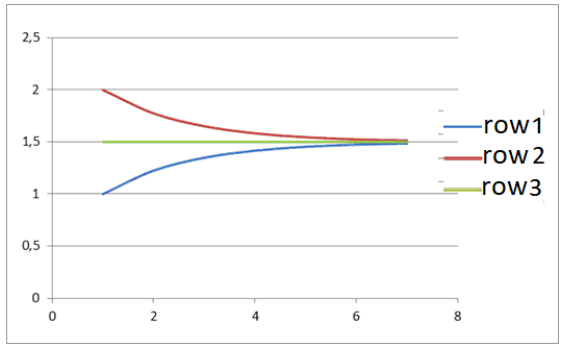

Figure 4. Equilibrium price

If $p(0) = 2, p(0) > p^*$, then $C = -0{,}5 \Rightarrow p(t) = 1{,}5 - 0{,}5 \cdot e^{-6t}$. The price of a good rises towards the equilibrium price. (row 1, Figure 4). If, then $C = 0{,}5 \Rightarrow p(t) = 1{,}5 + 0{,}5 \cdot e^{-6t}$. The price of a commodity decreases, approaching the equilibrium price (row 2 in Figure 4). So, we have shown that the price varies depending on the relationship between supply and demand. The price increase is directly proportional to the excess of demand over supply and the duration of this excess.

Conclusions. The use of mathematical modeling in economics makes it possible to make quantitative economic analysis deeper, expand the scope of economic information, and intensify economic calculations. We considered the economic model of Evans to study the establishment of an equilibrium price in the market for one product. It is different in nature from the original. The study of the properties of the original using a mathematical model

is more convenient, cheaper, and takes less time than physical modeling. The model was solved using the apparatus of differential equations. Graphs of the dependence of price on time are constructed, which prove the main assumption of the model that the price changes depending on the relationship between supply and demand and its increase is directly proportional to the excess of demand over supply and the duration of this excess. The application of the method of mathematical modeling in the economy is an objective stage in its development, associated with the existence of stable quantitative patterns and the possibility of a formalized description of many, although by no means all, economic processes.

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