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# МАТЕМАТИЧНІ МЕТОДИ, МОДЕЛІ ТА ІНФОРМАЦІЙНІ ТЕХНОЛОГІЇ В ЕКОНОМІЦІ

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## MATHEMATICAL MODELING OF THE MARKET OF THREE GOODS IN TERMS OF SUPPLY LAG

### МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ РИНКУ ТРЬОХ ТОВАРІВ В УМОВАХ ЛАГА ПОСТАЧАННЯ

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*The mathematical model of the free market of three goods in the conditions of supply lag in the interval of discrete time with market parameters, demand vector and initial conditions is considered. The results of simulation modeling of the market of 3 goods with the values of parameters and initial conditions are obtained. Graphs of the optimal dynamics of prices of three competing goods, graphs of the dynamics of demand for the three competing goods at the optimal dynamics of prices are analyzed. Graphically presents the dynamics of additional supplies of three competing products, the order of which was made  $\tau$  steps earlier in forecasting at time  $t$  optimal prices. It was found that during the initial time interval the balances of unsold goods accumulate, the market is overstocked, so at the end of the interval orders for goods are terminated. The dynamics of the optimal allowable total deliveries of goods to the market and the graphs of the dynamics of sales of goods are presented and analyzed. The dynamics of the maximum allowable profit of the seller is obtained.*

**Key words:** mathematical model, demand function, supply function, equilibrium, delivery lag, initial conditions.

У роботі досліджено граничні (потенційні) можливості отримання максимального сукупного доходу продавця на вільному ринку трьох товарів із подальшим синтезом оптимальної детермінованої стратегії постачання товару на ринок. Ринковий процес складається з багатьох актів обміну товарами та послугами, у яких беруть участь продавець, на стороні якого є пропозиція товару, і покупець, представлений попитом на товар. Розглянуто математичну модель вільного ринку трьох товарів в умовах лага постачання на інтервалу дискретного часу з параметрами ринку, вектором попиту та початковими умовами. За допомогою математичної моделі отримано результати імітаційного моделювання ринку трьох товарів зі значеннями параметрів та початкових умов. Представлено графік оптимальної динаміки цін трьох конкуруючих товарів, початкові ціни двох з яких (1-го та 2-го) нижчі за рівноважні, а третього – вищі за рівноважну. Згодом ціни всіх трьох товарів наближаються до асимптотично рівноважних значень (до точки спокою). Також представлено графік динаміки попиту на розглянуту трійку конкуруючих товарів за оптимальної динаміці цін. Це один із результатів впливу один на одного конкуруючих товарів. Продемонстровано графіки динаміки додаткових постачань трійки конкуруючих товарів, замовлення яких зроблено  $\tau$  кроками раніше під час прогнозування на момент часу

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*t* оптимальних цін. Виявлено, що протягом початкового інтервалу часу накопичуються залишки непроданих товарів, ринок затоварюється, тому наприкінці інтервалу замовлення товарів припиняються. Проаналізовано представлену динаміку оптимальних допустимих сумарних залишків та замовлень постачання товарів ринку та графіки динаміки продажів товарів. Отримано динаміку максимально допустимого прибутку продавця з огляду на оптимальну стратегію постачання товарів на ринок, ринок автоматично виходить зі стану кризи, викликаного порушенням його рівноваги, і переходить до рівноважного руху, асимптотичної рівноваги, що забезпечує максимальний прибуток продавця та повне задоволення споживчого попиту. Імітаційне моделювання ринку трьох товарів проілюструвало деякі основні закономірності взаємодії товарів на ринку майже вільної конкуренції.

**Ключові слова:** математична модель, функція попиту, функція пропозиції, рівновага, лаг поставок, початкові умови.

**Formulation of the problem.** The application of mathematical modeling in economics allows to make deeper quantitative economic analysis, expand the field of economic information, and make more efficient economic calculations. The mathematical model differs in nature from the original, but the study of the properties of the original using the mathematical model is more convenient, cheaper and less time consuming. The application of the method of mathematical modeling in economics is an objective stage of its development, associated with the existence of stable quantitative patterns and the possibility of a formalized description of many, though not all, economic processes.

**Analysis of recent research and publications.** Building a market model, modeling and forecasting its development is one of the most important problems of the economy. Most market models were built on the principle of establishing a competitive balance, the existence of which was stated in the work of Walras [1]. The mathematical substantiation of the Walras hypothesis was performed in the 1950s in the works of Arrow-Debre [2], Mackenzie, Gale, Nicaida. Further work was carried out to improve the models and their generalization in the monographs of Morishima, Nikaido, Lancaster and other modern authors [3; 4]. These market models struck a balance between supply and demand, but could not be a market model, as they, firstly, lacked competition between producers and consumers, and secondly, did not reflect the focus of market participants (producers and consumers), which is the basis of competition [5–7]. Imbalance characterizes a market that is not in equilibrium [8]. Imbalance can occur for a very short time or over a long period of time.

**Formulation of the goals of the article.** The market model reflects the balance between supply and demand, taking into account each market participant. A mathematical model has been built, which, along with the balance sheet, can reflect the purposefulness of each market participant [9]. Methods for solving a vector problem based on the normalization of criteria and the principle of guaranteed results have been developed [10–12]. In a competitive market with a large number of buyers and sellers, we use the method of search to determine the equilibrium price, excluding prices at which there is a surplus or shortage of product. It is necessary to investigate the behavior of the mathematical model in the case of three products, to analyze the results.

**Presenting main material.** Consider the example of the market  $n = 3$  goods with a supply lag  $\tau = 10$  at a discrete time interval from  $t = -\tau$  до  $t = T = 1000$  according to market parameters:

$$R = \begin{pmatrix} 50 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 50 \end{pmatrix}, P_1 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, P_2 = \begin{pmatrix} 0,3 \\ 0,2 \\ 0,1 \end{pmatrix}; \quad (1)$$

demand vector parameters:

$$Q_m = \begin{pmatrix} 8 \\ 4 \\ 2 \end{pmatrix}, A = \begin{pmatrix} 0,8 & -0,2 & -0,1 \\ -0,4 & 0,4 & -0,1 \\ -0,2 & -0,1 & 0,3 \end{pmatrix}; \quad (2)$$

all products are competing because all non-diagonal elements of matrix A are non-negative) and initial conditions:

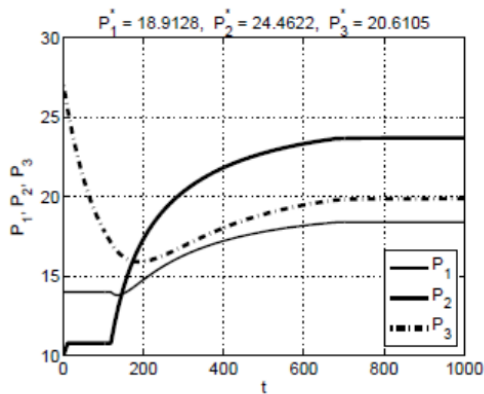
$$P(0) = P_0 = \begin{pmatrix} 14 \\ 10 \\ 27 \end{pmatrix}, \quad Q^o(0) = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}. \quad (3)$$

The main diagonal minors of the matrix A are positive:

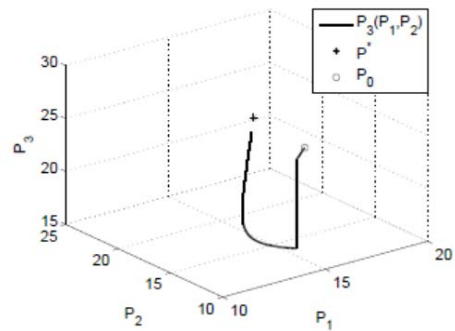
$$\det(0,8) = 0,8 > 0, \quad \det \begin{pmatrix} 0,8 & -0,2 \\ -0,4 & 0,4 \end{pmatrix} = 0,24 > 0, \quad \det \begin{pmatrix} 0,8 & -0,2 & -0,1 \\ -0,4 & 0,4 & -0,1 \\ -0,2 & -0,1 & 0,3 \end{pmatrix} = \det(A) = 0,048 > 0.$$

Thus, the matrix A is positive. Then the matrices  $A + A^T$  та  $A + A^T + R$  – positively defined, and accordingly there is an optimal equilibrium state of the market. In the picture 1–13 the results of simulation modeling of the market of 3 goods with the values of parameters and initial conditions in example are given. In the picture 1, 2 graphs of the optimal dynamics of prices of three competing goods are presented, the initial prices of two of which (1st and 2nd) are lower than equilibrium, and the third - higher than equilibrium. The graphs show that over time, the prices of all three goods are approaching asymptotically equilibrium values (to rest)  $P^*$ .

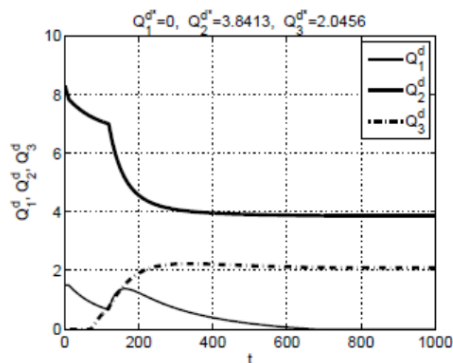
In the picture 3, 4 graphs of dynamics of demand for the considered three competing goods at optimum dynamics of prices are presented. It can be seen that stable asymptotic



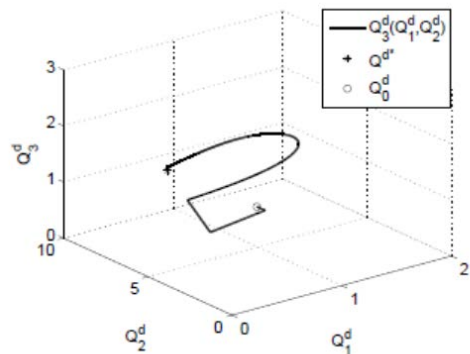
Drawing 1. Prices dynamics



Drawing 2. Phase trajectory of prices



Drawing 3. Dynamics of demand



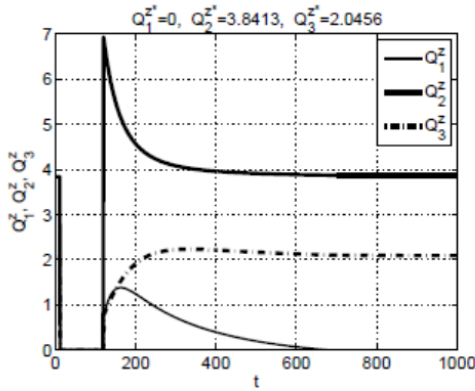
Drawing 4. Phase trajectory of demand

demand is achieved for the 2nd and 3rd commodities, while for the 1st commodity the demand asymptotically decreases to zero. This is one of the results of the influence of competing products on each other.

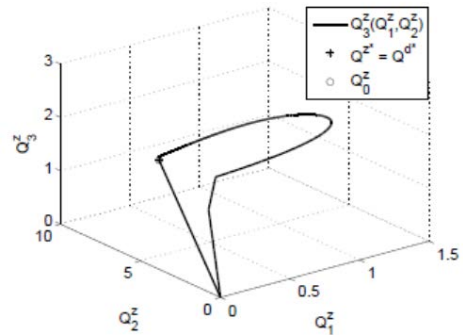
In the picture 5, 6 graphs of the dynamics of additional supplies of the analyzed three competing products are presented, whose order is made  $\tau$  steps earlier when forecasting at time  $t$  optimal prices. The graphs show, that during the initial period of time from 0 to  $\tau = 10$  the market receives standing orders made on the previous equilibrium interval of the market. As can be seen from the figure 7, 8 during this initial interval, the balance of unsold goods accumulates, the market is overstocked, so that at the end of this interval, orders for goods cease. Accumulated balances are sold off for a long time until there is a shortage of goods. The volume of orders increases sharply for the 2nd product and gradually increases for the 1st and 3rd products. The market begins an equilibrium movement to asymptotic equilibrium, in which demand for the first commodity falls to 0. Therefore, orders for this product at some point in time begin to decline and reach 0. Orders for the 2nd and 3rd goods reach a stationary level, which ensures asymptotic market equilibrium.

On the graphs of the figure 9, 10 the dynamics of the optimal allowable total (balances plus orders) supply of market goods is presented.

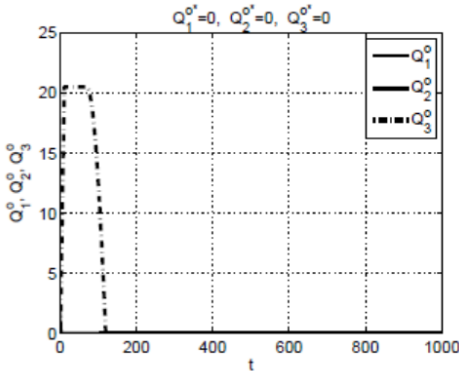
In the picture 11, 12 graphs of the dynamics of sales of goods are presented. It is clear that there is a certain time interval in which sales fall to zero. This happens when the 1st and 2nd goods, which are in effective demand, are not on the market due to market imbalances, and the



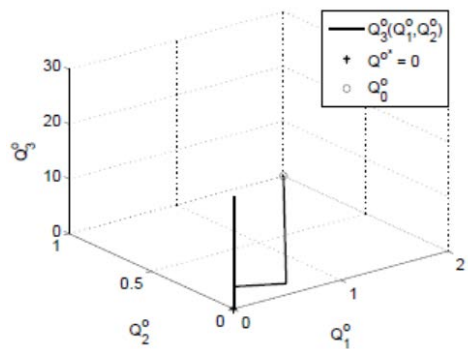
Drawing 5. Dynamics of orders



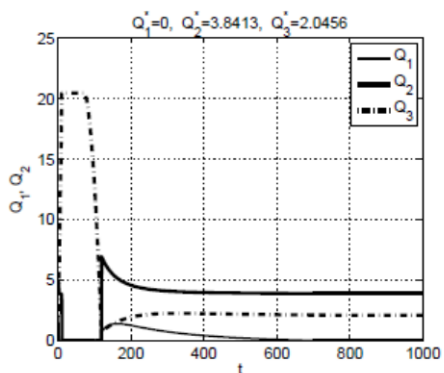
Drawing 6. Phase trajectory of orders



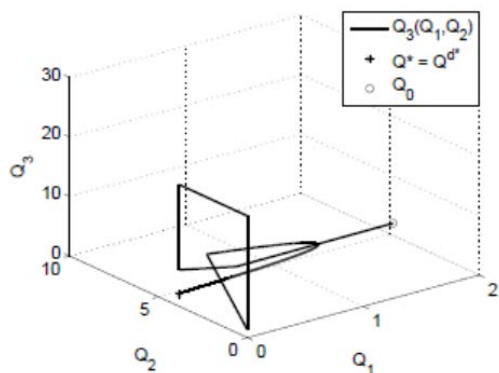
Drawing 7. Dynamics of balances



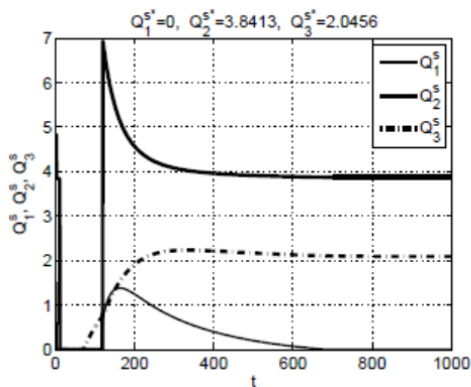
Drawing 8. Phase trajectory of residues



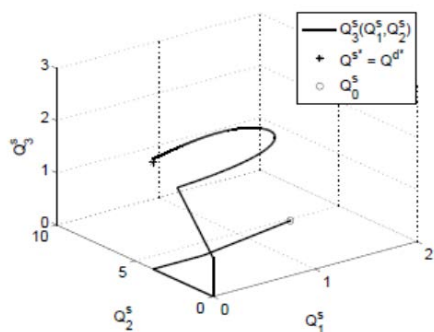
Drawing 9. Dynamics of supply



Drawing 10. Phase trajectory of supply



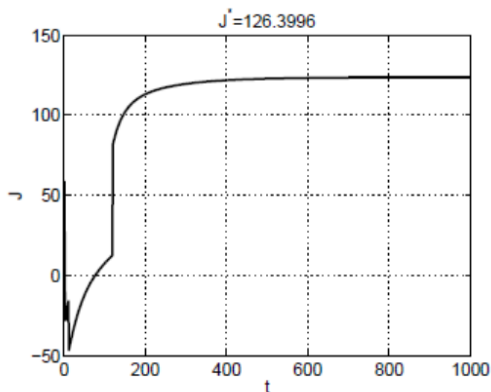
Drawing 11. Sales dynamics



Drawing 12. Phase trajectory of sales

3rd product for the same reason is so expensive that it becomes “out of pocket” buyers. Then the sales dynamics is restored, gradually approaching the asymptotically optimal level  $Q^*$ .

Eventually, in the picture 13 the dynamics of the maximum allowable profit of the seller is presented. It can be seen that as a result of bringing the market out of equilibrium, the



Drawing 13. Profit dynamics

total (for all goods) seller's profit falls to a negative value (losses exceed revenue), and by no means can the seller quickly restore the state of market profitability due to the restraining influence of three factors: limited effective customer demand, lag in the supply of goods and market inertia. Gradually, based on the optimal strategy of supplying goods to the market, the market automatically emerges from a state of crisis caused by its imbalance, and moves to equilibrium, to asymptotic equilibrium, which provides maximum profit for the seller and consumer demand.

**Conclusions.** The example of simulation modeling of the market of 3 goods illustrates some basic laws of interaction of goods in the market of almost free competition. "Almost", because the penalty component of the target function, which gives the market inertia, is an external factor that restricts free competition.

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